Physics 20

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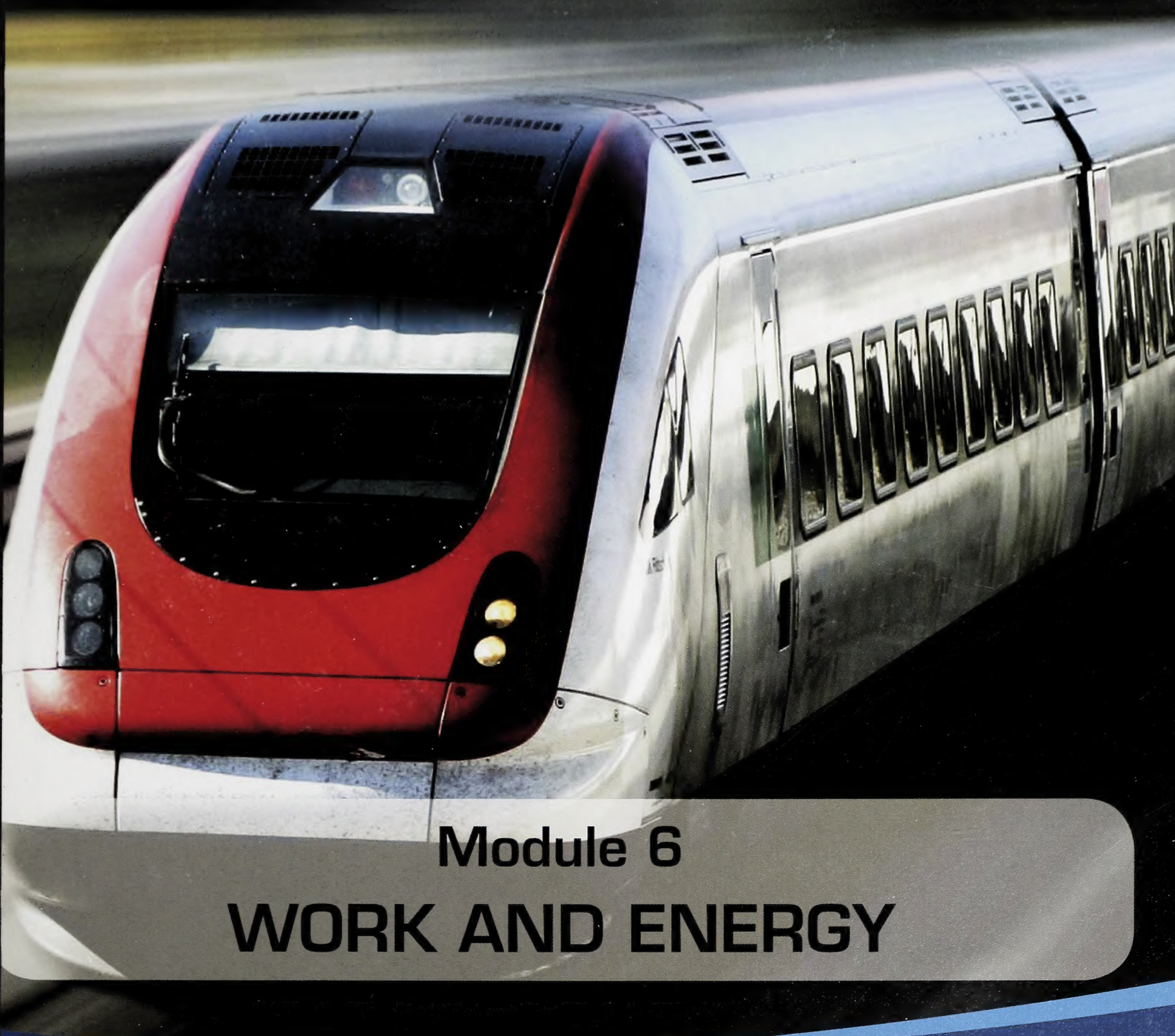

Module 6
WORK AND ENERGY

Rocky View School Division
Red Deer Catholic Regional Schools
Edmonton Public Schools
Pembina Hills Regional School Division No. 7
Black Gold Regional Schools
Peace Wapiti School Division No. 76
Calgary Board of Education
Alberta Education

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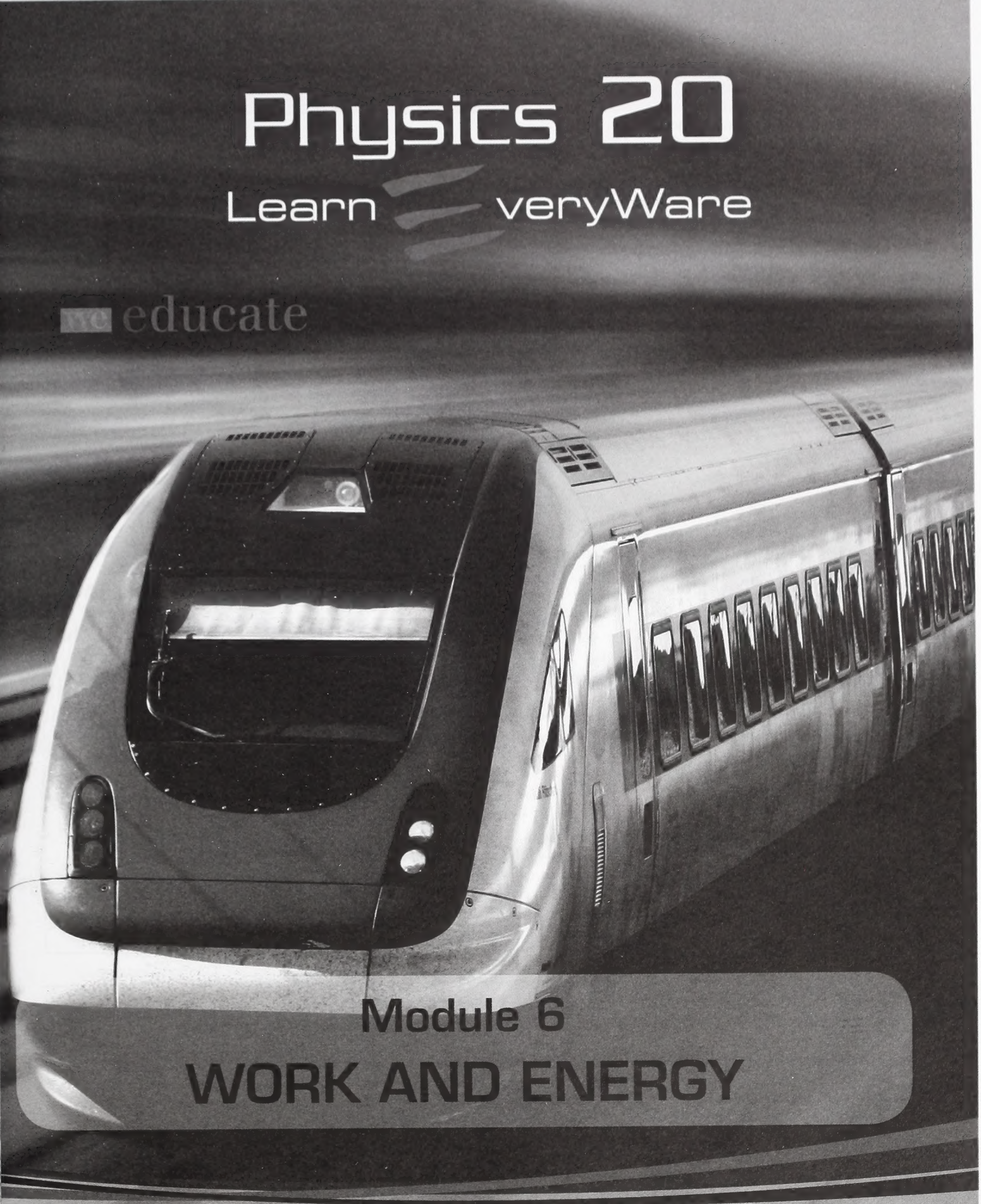

Module 6

WORK AND ENERGY

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PEMBINA HILLS

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PEACE WAPITI SCHOOL DIVISION NO. 76

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- Black Gold Regional Schools
- Calgary Board of Education
- Edmonton School District No. 7
- Peace Wapiti School Division No. 76
- Pembina Hills Regional Division No. 7
- Red Deer Catholic Regional Division
- Rocky View School Division No. 41

Physics 20
Module 6: Work and Energy
Student Module Booklet
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Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	

You may find the following Internet sites useful:

- Alberta Education, **http://www.education.gov.ab.ca**
- Learning Resources Centre, **http://www.lrc.education.gov.ab.ca**
- Tools4Teachers, **http://www.tools4teachers.ca**

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Contents

Module 6 —Work and Energy

Module Introduction

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In this module you will study work and its relationship to different forms of mechanical energies. You will explore the connection between work and power. You will analyze kinematics and dynamics problems that relate to the conservation of energy in an isolated system and examine the change in mechanical energy in a system that is not isolated.

The essential questions that you will be looking at in this module are the following:

- How does an understanding of conservation laws contribute to an understanding of the universe?
- How can mechanical energy be transferred and transformed?

Big Picture

Have you ever dreamed of soaring in the sky with the birds? It seems like such an effortless way to spend an afternoon.

You can't just jump into the air, though. You need to climb up to the top of the cliff with the necessary equipment, put on all of your equipment, and take a running leap over the edge. When you think about all of this, you realize that there is considerable effort involved before the fun of floating in the air ever begins.

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Just like you, the bird has to work hard to get up in the sky before it gets to use the air currents to keep it aloft.

We've all heard the saying "what goes up must come down." But for a while, both the bird and the person on the hang-glider seem to be breaking that rule. For a time, the hang-glider is breaking the law of gravity as he or she rises and circles far above the rocks at the bottom of the cliff. Something must be acting against the force of gravity, allowing the person with the hang-glider to float and maintain motion high above Earth.

It can be fun to work with gravity too. Can you imagine the adrenaline rush you would feel riding down a run on a bobsled at 125 km/h as you twist and turn and bump against the side of the track? Maybe you'd rather try ski jumping. How fast would you be going just at the edge of the jump? It's just you—and the abyss—and the 100 km/h gained from sliding down the long ramp toward the edge of the jump. Then, you fly. If you're really good, you might land 140 m away and win an Olympic gold medal.

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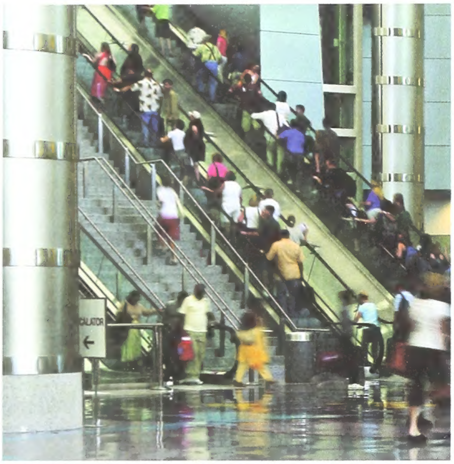
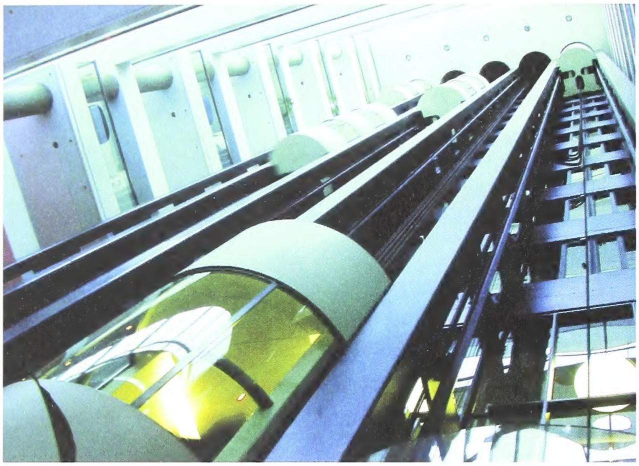
There are times when you fight against gravity to rise to higher levels. Think about the bobsled you rode down the track at 125 km/h. Would you want to carry your sled with you as you climb back up to the top of the track? In some cases, it's just not practical to carry objects up by hand. Think about the building materials lifted by the crane shown in the photo. Would you want to carry these building materials to the top of the building?

Climbing up a hill or up a flight of stairs is something people can do, but they often use machines instead. When you are in a mall, do you climb the stairs to the second level, or do you take an escalator or elevator? Is it because you really don't want to walk up the stairs, or is it because you can't find any stairs to use?





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Perhaps you would like to try a sport where falling is balanced with rising, and it feels like some of the effort you put in is returned.

If so, a trampoline would be to your liking. Each time you fall, it feels like you are being shot back up into the air. The trampoline debuted as an Olympic sport in 2000, and the athletes that compete at this level really do look like they are defying gravity as they effortlessly perform twists, turns, and somersaults in midair.

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As you work through this module, keep the following questions in mind. They will help you understand the relationship between different forms of energy and work.

- What is work?
- What is gravitational and elastic potential energy?
- What is kinetic energy?
- How do gravitational potential energy, elastic potential energy, and kinetic energy interact?
- Is mechanical energy conserved in isolated (frictionless) and non-isolated (friction-filled) environments?
- What effect does the presence of friction have when converting potential energy into kinetic energy?
- What effect does work have on mechanical energy?
- How is the work done by a machine calculated?
- What effect do conservative and non-conservative forces have on the mechanical energy of lifting machines?

- What is the work-potential energy theorem?
- What is power?
- What is the relationship between power and work?
- How is efficiency determined?

In This Module

Lesson 1—Work, Potential Energy, and Kinetic Energy

This lesson explores work and energy.

You will explore the following questions:
- What is work?
- What is gravitational and elastic potential energy?
- What is kinetic energy?
- How do gravitational potential energy, elastic potential energy, and kinetic energy interact during a bungee jump?

Lesson 2—Conservation of Mechanical Energy

In this lesson you will examine mechanical energy and the law of conservation of energy.

You will explore the following essential questions:
- Is mechanical energy conserved in isolated (frictionless) and non-isolated (friction) environments?
- What effect does friction have when converting potential energy into kinetic energy?
- What effect does work have on mechanical energy?

Lesson 3—Mechanical Energy and the Work-Energy Theorem

An examination of the relationship between work and energy is the focus of this lesson.

You will explore the following essential questions:
- How do you know how much work is done in a given situation?
- What effect do conserved and non-conserved forces have on the gain in mechanical energy?
- What is the work-potential energy theorem?

Lesson 4—Work and Power

To create greater efficiencies in society, we must first understand the connection between work and power. This lesson will explore the concept of power as it relates to work.

You will explore the following essential questions:
- What is power?
- What is the relationship between power and work?
- How is efficiency determined?

Module 6 Assessment

The assessment for Module 6 consists of four (4) assignments:
- Module 6: Lesson 1 Assignment
- Module 6: Lesson 2 Assignment
- Module 6: Lesson 3 Assignment
- Module 6: Lesson 4 Assignment

In addition, you will be expected to choose one of the Reflect on the Big Picture items from this module and share it with your teacher.

Lesson 1—Work, Potential Energy, and Kinetic Energy

Get Focused

The first modern bungee jump was made on April 1, 1959, from the 76-m Clifton Suspension Bridge in Bristol, England. The jumpers were arrested shortly after for performing what was considered a very dangerous stunt. Today, bungee jumping is commonplace, with thrill seekers leaping from crane platforms, hot-air balloons, tall buildings, and bridges.


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Although there have been several million successful, safe jumps, there have been some fatalities and serious injuries. Leaping from tall buildings and bridges is inherently dangerous. But with the right equipment, careful calculations, and proper fittings, the risk of death and injury can be minimized.

The most common safety issues involve securing the elastic bungee to the jumper and correctly calculating the length of elastic bungee to be used, based on the weight of the jumper and the height of the jump. Failing to do either of these tasks correctly could lead to injury or death for the jumper.

You may correctly assume that a heavier jumper will require a shorter bungee cord since he or she will stretch the cord farther than someone who is lighter. But you will likely incorrectly assume that when the cord reaches its natural length (as if no mass were attached to it), the jumper will begin to slow down. In fact, the jumper will continue to speed up. To understand this, you need to consider the origin of the energy and the energy transfers that take place between the bungee cord and the jumper as he or she bobs up and down within Earth's gravitational field.

In this lesson you will explore the following questions:

- What is work?
- What are gravitational potential energy and elastic potential energy?
- What is kinetic energy?
- How do gravitational potential energy, elastic potential energy, and kinetic energy interact?

Module 6: Lesson 1 Assignments

Your Lesson 1 Assignment in the Module 6 Assignment Booklet requires you to submit a response to the following:

- Try This—TR 1, TR 2, TR 3, TR 4, TR 5, TR 6, and TR 7
- Lab—LAB 1, LAB 2, LAB 3, LAB 4, and LAB 5

The other questions in this lesson are not marked by the teacher; however, you should still answer these questions. The Self-Check and Try This questions are placed in this lesson to help you review important information and build key concepts that may be applied in future lessons.

After a discussion with your teacher, you must decide what to do with the questions that are not part of your assignment. For example, you may decide to submit to your teacher the responses to Try This questions that are not marked. You should record the answers to all the questions in this lesson and place those answers in your course folder.

Explore


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Bungee jumping is a **mechanical system**, which is defined by having both potential energy and **kinetic energy**. Potential energy is the energy of an object that is related to its position—it is also referred to as stored energy. Kinetic energy is the energy of an object related to its motion. The sum of the potential energy and the kinetic energy in a system (such as a bungee cord and jumper) is known as mechanical energy. In order to understand this, you must first understand that energy is not created or destroyed, it is only transferred.

mechanical system: a system that has both potential and kinetic energy

kinetic energy: energy that a body has because of its motion

Work

Technically, bungee jumping starts from the ground. The jumper either climbs a hill to access a bridge deck or is raised up on a platform using a crane. This process involves work that is done against the force of gravity on the surface of Earth. The concept of work must be carefully defined and understood—it has different meanings for different applications. In physics, work is described as the product of force and displacement—vector quantities that have direction.

It is important to note the language that is used in physics to describe work. In physics, we talk about the amount of work that is done "on" an object. For example, if you were to stand still and hold up a 40.0-kg object, you would be using energy; therefore, you would assume that you are doing work on the object. However, this work is not done on the object. Work is done on your muscles as you force them to contract, so no work is actually done on the object, and it does not gain or lose any energy as a result.

If the object does not move through some displacement, there is no work done. In a similar way, if there is no force acting on the object, there is no work done on the object.

Recall from Science 10 that in order to do work on an object, there must be an applied force (which could be applied to overcome a force, such as gravity or friction). The object must also move in the same direction as the applied force (or a component of the applied force).

Work is a measure of the amount of energy transferred when a force acts over a given displacement. It is the product of the magnitude of the applied force and the magnitude of the displacement of the object in the direction of that force. Expressed as an equation, it is

$$W = F_{\parallel}\Delta d$$

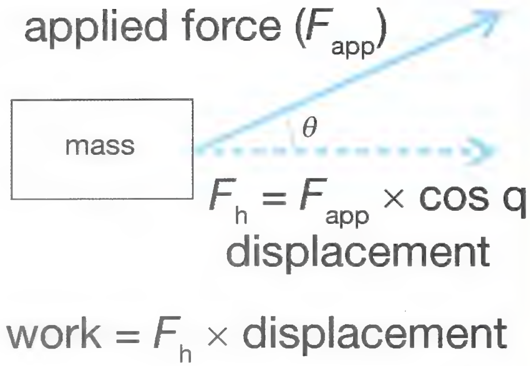
This is often written as $W = F\Delta d$.

Quantity	Symbol	SI Unit
work	W	joule (J or N•m)
force (the component of the force that is parallel to the displacement of the object)	$F_{\parallel}$	newton (N)
magnitude of displacement	d	metre (m)

1 J is equal to 1 N•m. Work is the product of the magnitudes of two vector quantities. Hence, it is a scalar quantity.

Component analysis is used to find the work done when the force is not in the same direction as the displacement. In such cases, the component of the force that is parallel to the displacement must be calculated before you can determine the work done.

In this diagram, the angle between the F_{app} (applied force) and Δd (the direction of displacement) is θ; so, $F_{\parallel}$ (the force parallel to direction of displacement, which in this diagram is the horizontal force F_h) is calculated as $(F_{app} \cdot \cos\theta)$. Substituting into the original equation, you get the following:



$$\text{work} = (F_{app} \cdot \cos\theta)\,\Delta d$$

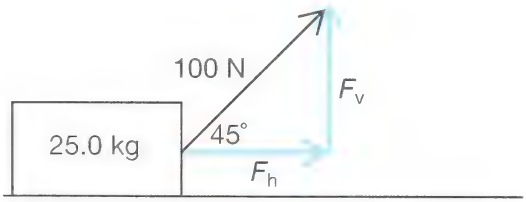
Read

Read pages 292 to 294 in your textbook to see examples of how this is applied. In "Example 6.1" on page 294, a horizontal force is pushing an object up a ramp.

The following example shows another situation where work is calculated, but this time the displacement is horizontal and the force is applied at an angle.

Example Problem 1

A 25.0-kg box is pulled 12.0 m along a level surface by a rope. If the rope exerts a force of 100 N at an angle of 45.0° to the horizontal, how much work is done on the box?



Given

$F = 100$ N $\quad m = 25.0$ kg $\quad \Delta d = 12.0$ m $\quad \theta = 45.0°$

Required

the work done on the box (W)

Analysis and Solution

The component of the applied force parallel to the displacement will have to be calculated using the cosine function, and used in the work formula.

$$
\begin{aligned}
W &= F_{\parallel}\Delta d \\
&= \left(F\cos 45.0^{\circ}\right)\Delta d \\
&= (100\ \text{N})\left(\cos 45.0^{\circ}\right)(12.0\ \text{m}) \\
&= 849\ \text{J}
\end{aligned}
$$

Paraphrase

There were 849 J of work done on the box.

In the next example, the force must be calculated using concepts and formulas learned in previous modules in order to find the work done.

Example Problem 2

A 500-kg motorcycle is travelling at 36.0 km/h and is brought to a stop after skidding 50.5 m. What work did the frictional force do?

Given

$m = 500$ kg $\quad v_i = 36.0$ km/h $\quad v_f = 0.00$ km/h $\quad \Delta d = 50.5$ m

Required

the work done by friction to stop the motorcycle (W)

Analysis and Solution

The force of friction and the displacement are parallel, so no angle is required. Force is not given, so it must be calculated using a kinematic formula and Newton's second law. The first step is to convert the initial speed to m/s.

$$\frac{36.0\ \text{km}}{1\ \text{h}} \times \frac{1000\ \text{m}}{1\ \text{km}} \times \frac{1\ \text{h}}{3600\ \text{s}} = 10.0\ \text{m/s}$$

Use a kinematic formula to find the magnitude of the acceleration.

$$v_f^2 = v_i^2 + 2ad$$
$$a = \frac{v_f^2 - v_i^2}{2d}$$
$$a = \frac{(0.00\text{ m/s})^2 - (10.0\text{ m/s})^2}{2(50.5\text{ m/s})}$$
$$a \doteq 0.990\ 099\text{ m/s}^2$$

Use Newton's second law to find the magnitude of the net force.

$$\vec{F}_{net} = m\vec{a}$$
$$F = (500\text{ kg})\left(0.990\ 099\text{ m/s}^2\right)$$
$$F = 495.0\text{ N}$$

Calculate the work.

$$W = F\Delta d$$
$$W = (495\text{ N})(50.5\text{ m})$$
$$W = 2.50 \times 10^4\text{ J}$$

Paraphrase

The frictional force did 2.50×10^4 J of work.

Module 6: Lesson 1 Assignment

Remember to submit the answers to TR 1, TR 2, TR 3, and TR 4 to your teacher as part of your Lesson 1 Assignment.

Try This

TR 1. A 2.20-N object is held 2.20 m above the floor for 10.0 s. How much work is done on the object?

TR 2. A 60.0-kg student runs at a constant velocity up a flight of stairs. If the vertical distance of the stairs is 3.2 m, what is the work done against gravity?

Hint: Use the formula $F_g = mg$.

TR 3. A 10.0-kg object is accelerated horizontally from rest to a velocity of 11.0 m/s in 5.00 s by a horizontal force. How much work is done on this object?

Hint: Use the following formula:

$$\left(a = \frac{v_f - v_i}{\Delta t}\right)$$

TR 4. A 90.0-N box is pulled 10.0 m along a level surface by a rope. If the rope makes an angle of 20.0° with the surface, and the force exerted through the rope is 75.0 N, how much work is done on the box?

Self-Check

SC 1. A 25.0-kg object is accelerated from rest through a distance of 6.0 m across a level floor in 4.0 s. If the force due to friction between the object and the floor is 3.8 N, what is the work done in moving the object? (**Note:** $F_{net} = ma$, where a is the observed acceleration and $F_{net} = F_{app} + F_f$, where F_f is in the opposite direction of F_{app}.)

Check your work with the answer in the appendix.

When a bungee jumper is lifted at constant velocity via a crane platform, work is done against gravity, causing the jumper to have a given height relative to the ground below. In such a case, the lifting force applied to the jumper opposes his or her weight. Substituting weight (F_g) and vertical displacement (h) into the work equation produces a new equation for describing the work done in lifting an object against Earth's gravitational field.

work = force × displacement
work = weight × vertical displacement
$W = mg\Delta h$

It is commonly accepted that if the object is lifted from the ground to a given height, the equation may be written as $W = mgh$. However, care should be taken to ensure that it is the change in height that is substituted into the equation.

Assuming that all of the work is done on the object being lifted vertically, the object gains an equal amount of **gravitational potential energy**. Therefore,

gain in potential energy = work done to lift it
$E_p = mg\Delta h$ (or $E_p = mgh$, where h is the change in height)

gravitational potential energy: the energy of an object due to its height above Earth

Expressed as an equation, it is

$E_p = mgh$

Quantity	Symbol	SI Unit
gravitational potential energy	E_p	J
mass	m	kg
acceleration due to gravity	g	m/s^2
height	h	m

1 J is equal to 1 N•m. Energy is a scalar quantity.

Gravitational potential energy is one form of potential energy involved in the bungee jumper-and-cord system. A second form of energy, called **elastic potential energy**, is also involved.

elastic potential energy: energy stored in a spring or elastic object that has been compressed or stretched

Read

Read about gravitational potential energy on pages 295 and 296 of your textbook.

Self-Check

SC 2. Solve question 1 of "Practice Problems" on page 296 of your textbook.

Check your work with the answer in the appendix.

Read

Read about the importance of the reference point used on pages 297 and 298 of your textbook.

Self-Check

SC 3. Solve question 3 of "Practice Problems" on page 298 of your textbook.

Check your work with the answer in the appendix.

Elastic Potential Energy

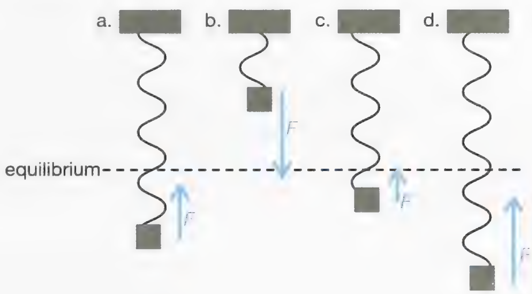
When a person attached to a bungee cord jumps from the top of a crane, he or she immediately reduces his or her height and, hence, the amount of gravitational potential energy. Where does the energy go? Obviously, the potential energy becomes kinetic (motion) energy. But in what form is the energy when the person comes to a complete stop before impacting the ground?

Hooke's law: the amount of stretch (deformation) of an elastic object is proportional to the force applied to deform it

Expressed as an equation, it is

$F = kx$

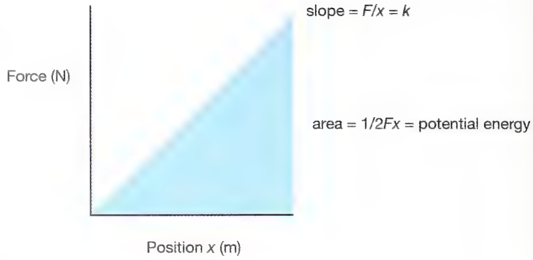
A bungee cord is a giant spring, and it holds the energy as elastic potential energy. In principle, a bungee jumper is identical to a spring with a weight hanging from it—which exhibits simple harmonic motion. This means that the spring is vibrating with a constant frequency or period of motion and that there is a restoring force directed towards the central equilibrium point. Furthermore, the magnitude of the restoring force is proportional to the displacement from the equilibrium point. This is known as **Hooke's law**.



Quantity	Symbol	SI Unit
force (magnitude of applied force)	F	N
force constant of the spring (often called elastic constant or spring constant)	k	N/m
amount of stretch (deformation) of the spring	x	m

Graphing the force required to compress a spring versus the length of the spring gives a slope that is equal to the spring constant.

In Newtonian mechanics, $W = Fd$; but in the case of a spring, the force is not constant. So, the graphical version of Hooke's law is used to generate an expression for the elastic potential energy of a weighted spring. Taking the area of the force-position graph for a spring reveals that $E_p = \frac{1}{2}Fx$.



Combining two equations will give an energy equation that can be used to solve problems in terms of the spring constant and the amount of stretch or deformation in the spring.

$$F = kx$$
$$E_p = \frac{1}{2}Fx$$
$$E_p = \frac{1}{2}(kx)x$$
$$E_p = \frac{1}{2}kx^2$$

The force of gravity and the elastic force of an ideal spring are referred to as conservative forces. For a conservative force, work done against it is stored and is recoverable. If the work is done against gravity, as is the case with bungee jumping, this work is stored and transferred back to gravitational potential energy and kinetic energy as the jumper bounces back up.

It must be noted that friction is a non-conservative force. Work done against friction cannot be recovered. Work done against friction is converted into some other form of energy—mainly thermal energy. In the end, the bungee jumper stops bouncing as all the energy is eventually converted to heat by non-conservative forces, such as friction in the bungee cord and air friction.

Read

Read "Hooke's Law" and "Elastic Potential Energy" on pages 299 to 302 in your textbook.

Self-Check

SC 4. Solve question 5 of "Practice Problems" on page 301 of your textbook.

Check your work with the answer in the appendix.

Kinetic Energy

kinetic energy: energy that a body has because of its motion

Expressed as an equation, it is

$$E_k = \frac{1}{2}mv^2$$

From the moment the jumper steps off the bridge or platform until the bungee cord starts to stretch, the jumper experiences free fall (weightlessness, assuming no air resistance). During this time period, the jumper's gravitational potential energy is converted into **kinetic energy** (energy of motion). He or she accelerates downward at the rate of gravity, gaining considerable speed and kinetic energy until the bungee cord stretches far enough that the restoring force exceeds the force of gravity. At that point, the net acceleration reverses direction, slowing the jumper down until he or she comes to a momentary stop at the bottom of the jump.

Quantity	Symbol	SI Unit
kinetic energy	E_k	J
mass	m	kg
speed	v	m/s

The kinetic energy of an object is equal to the work done to accelerate it.

Read

Read "Kinetic Energy" on pages 302 to 304 in your textbook.

Self-Check

SC 5. Solve question 1 of "Practice Problems" on page 303 of your textbook.

SC 6. Solve question 1 of "Practice Problems" on page 304 of your textbook.

Check your work with the answer in the appendix.

Module 6: Lesson 1 Assignment

Remember to submit the answers to TR 5, TR 6, and TR 7 to your teacher as part of your Lesson 1 Assignment.

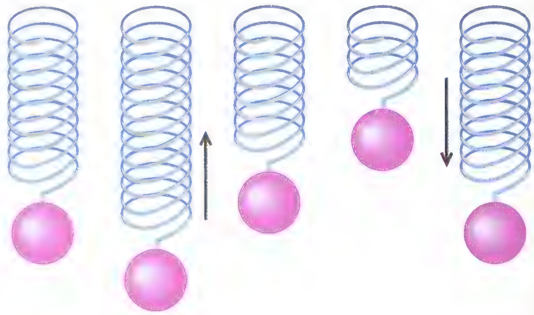
TR 5. A 25.0-N object is held 2.10 m above the ground. What is the potential energy of the object with respect to the ground?

TR 6. The kinetic energy of a 20.0-N object is 5.00×10^2 J. What is the speed of this object?

TR 7. A 10.0-N object is accelerated uniformly from rest at a rate of 2.5 m/s^2. What is the kinetic energy of this object after it has accelerated a distance of 15.0 m?

Lesson 1 Lab: Energy Interactions

This lab lets you simulate the simple harmonic motion of a weighted spring. You can graphically explore changes in the weighted spring's potential and kinetic energy, velocity, and position over time. You can vary the spring constant, mass, and amplitude.



Problem

How do gravitational potential energy, elastic potential energy, and kinetic energy interact during a bungee jump?

To visualize this process, you will use a simulation of a weighted spring. Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms "energy conservation" in the search bar. Choose the item called "Energy Conservation: Weighted Spring."Start thesimulation; then continue with the Procedure and Observations section. You can learn more about the simulation and how to use it by reading the Show Me found at the top of the simulation screen.

Procedure and Observations

On the simulation, click and drag the object up and down. Observe the red and blue display for the potential and kinetic energy of the object.

Module 6: Lesson 1 Assignment

Remember to submit the answers to LAB 1, LAB 2, and LAB 3 to your teacher as part of your Lesson 1 Assignment.

LAB 1. Drag the object to the maximum height. What form is all of the energy in at this position? Is it gravitational potential energy or both gravitational energy and elastic potential energy? Explain your answer.

LAB 2. Where is the object located, relative to the equilibrium position, when it has the greatest kinetic energy? Explain why this occurs at this position.

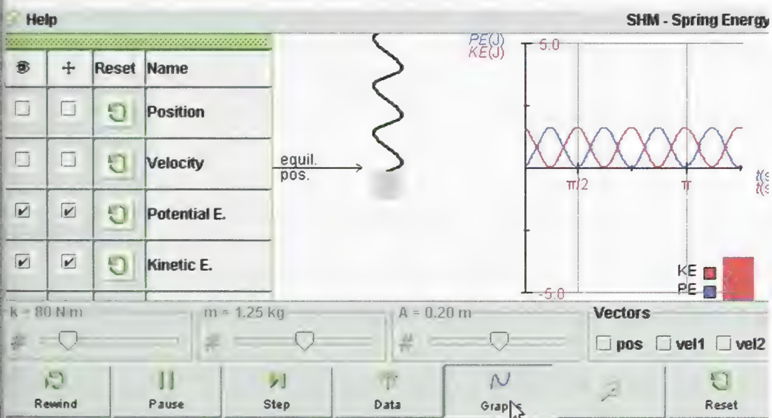
LAB 3. What form of energy is present at the lowest position? Explain how this energy is stored differently than the energy stored at the highest position.

Module 6: Lesson 1 Assignment

Remember to submit the answers to LAB 4 and LAB 5 to your teacher as part of your Lesson 1 Assignment.

Conclusion

Select the "Graphs" button, and turn on the display for both the potential energy and kinetic energy graphs. Press "Play," and observe the graphs in real time as the object bounces.



LAB 4. Explain why the potential and kinetic energies vary, as displayed in the graph. Select the mechanical energy graph for a hint!

LAB 5. Explain how gravitational potential energy, elastic potential energy, and kinetic energy interact during a bungee jump. You may use diagrams as part of your explanation.

Reflect and Connect

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Bungee jumping is energy artwork—a periodic dance of energy transfers that repeats itself with each trip up and down. The jumper starts with gravitational potential energy at the platform, and then jumps, experiencing a free fall and a dramatic increase in kinetic energy with a similar decrease in potential energy. Once past the equilibrium position, the bungee cord absorbs the kinetic energy, converting it to elastic potential energy as the jumper approaches the lowest point, where the process reverses. On the way up, elastic potential energy becomes kinetic energy, which is then converted back to gravitational potential energy as the jumper approaches the maximum height. And then, it all reverses and repeats. The process does not continue forever because the bungee cord and jumper are an open system, letting energy out in the form of heat and sound.

Consider another thrill ride based on the same principles, but in reverse. On this ride, a cage is held up by several bungee cords. To start the ride, the cage is pulled down from a higher equilibrium position, a rider is loaded, and the cage is released, surging upward with a large acceleration.

Can you describe the energy transfers that occur on this ride and explain the observed acceleration?

Store your completed reflection in your Physics 20 course folder.

Discuss

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Have you ever been on a trampoline? Trampolines are a potential source of fun, but they are also a potential source of injury. Have you ever been double-bounced on a trampoline? Imagine you are jumping on a trampoline, a process that is energetically similar to that of a bungee jump. Now, introduce a second person of equal mass who is also jumping up and down.

In the discussion forum, explain how one jumper could receive the energy of both jumpers, creating an opportunity for injury. Relate this to your own personal experiences on a trampoline, if possible.

For information about trampoline safety, research the Internet. There are trampoline safety tips at the Health Canada website.

Reflect on the Big Picture

Each of the Reflect on the Big Picture sections in this module will ask you to consider gravitational, frictional, or elastic forces. To help you consolidate your learning from this lesson, complete at least one of the following activities.

- Think about your experiences to find examples of elastic forces that you have been involved with, such as archery, pogo sticks, paddle balls, bungee jumps, trampolines, and rubber bands. Prepare a short presentation about two or three of your experiences. Identify what work is done, and explain the transfer of energy that occurs in the activity. You can present it in writing, orally, or in a video or multimedia format.

- The bungee jump that introduced this lesson is a spectacular example of elastic and gravitational forces being applied simultaneously. Prepare and label a sequence of three or four illustrations showing the changes in gravitational potential energy, elastic potential energy, and kinetic energy as the bungee jumper falls from the platform to the jumper's lowest point.

Store your completed reflection in your Physics 20 course folder.

Module 6: Lesson 1 Assignment

Make sure you have completed all of the questions for the Lesson 1 Assignment. Check with your teacher about whether you should submit your assignment now or wait until all of the Module 6 assignments have been completed.

Lesson Summary

In this lesson you explored the following questions:
- What is work?
- What are gravitational potential energy and elastic potential energy?
- What is kinetic energy?
- How do gravitational potential energy, elastic potential energy, and kinetic energy interact during a bungee jump?

Work is a measure of the amount of energy transferred when a force acts over a given displacement. It is the product of the magnitude of the applied force and the displacement of the object in the direction of that force.

Gravitational potential energy is the energy of an object due to its position above the surface of Earth.

Elastic potential energy is energy produced or consumed by an object that is altered from its standard shape without permanent deformation.

Kinetic energy is the energy due to the motion of an object.

A bungee jump starts with gravitational potential energy. On the way down, the energy is converted to kinetic energy and, finally, to elastic potential energy at the very bottom. On the way up, the process is reversed. With each bounce, some energy is lost from the system, and eventually the motion stops.

Lesson Glossary

elastic potential energy: energy stored in a spring or elastic object that has been compressed or stretched

gravitational potential energy: the potential energy of an object due to its height above Earth

Hooke's law: the amount of stretch (deformation) of an elastic object is proportional to the force applied to deform it

kinetic energy: energy that a body has because of its motion

mechanical system: a system that has both potential and kinetic energy

work: the energy transferred by a force to a moving object; the product of a force and the distance through which the force is applied

Lesson 2—Conservation of Mechanical Energy

Get Focused

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Lake Louise, Alberta, has become a regular stop on the World Cup Ski Racing circuit. During these races, skiers generally come down the course at speeds ranging from 88–96 km/h, and they try to maintain maximum speed throughout the race. What factors should the skiers consider in trying to achieve very high speeds? At the starting gate, each racer has exactly the same amount of potential energy (assuming they have the same mass).

This means that the race is really about which skiers can convert more gravitational potential energy into kinetic energy as they move down the hill. In the best case scenario, all the potential energy will become kinetic. In reality, this doesn't happen since considerable friction forces will be acting on the skier. What effect does friction have on the conversion of potential energy into kinetic energy as the skier descends along the course?

In this lesson you will explore the following questions:

- Is mechanical energy conserved in isolated (frictionless) and non-isolated (friction) environments?
- What effect does the presence of friction have when converting potential energy into kinetic energy?
- What effect does work have on mechanical energy?

Module 6: Lesson 2 Assignment

Your Lesson 2 Assignment in the Module 6 Assignment Booklet requires you to submit a response to the following:

- Lab—LAB 2, LAB 3, LAB 4, LAB 6, and LAB 7
- Try This—TR 1, TR 2, and TR 3

You must decide what to do with the questions that are not marked by the teacher.

Remember that these questions provide you with the practice and feedback that you need to successfully complete this course. You should respond to all the questions and place those answers in your course folder.

Explore

Mechanical Energy

When a skier is moving down the slopes of a mountain, he or she has two forms of energy—potential and kinetic. The sum of the potential energy and the kinetic energy is called **mechanical energy**. Expressed as an equation, it is as follows:

mechanical energy: the sum of potential energy and kinetic energy

$E_m = E_p + E_k$

Quantity	Symbol	SI Unit
mechanical energy	E_m	J
gravitational and/or elastic potential energy	E_p	J
translational kinetic energy	E_k	J

One of the most general principles in physics is the principle of energy conservation. The **law of conservation of energy** states that within an isolated system, energy cannot be created or destroyed, but it can be converted from one form to another. Therefore, the sum of all of the energies in the system remains constant.

law of conservation of energy: a principle that states within an isolated system, energy cannot be created or destroyed, but it can be converted from one form to another

A special case of this principle, the law of mechanical energy conservation, states that the energy of an isolated mechanical system is conserved. An isolated system is one in which no energy enters or leaves. In such a case, the mechanical energy is conserved, so any change in potential energy is accompanied by an equal but opposite change in kinetic energy. Is a downhill ski racer an isolated system that converts gravitational potential energy completely into kinetic energy? If not, does a downhill skier obey the law of mechanical energy conservation?

You will now use a simulation to investigate the law of mechanical energy conservation, and you'll apply this to what is observed in a downhill ski race.

Lesson 2 Lab: The Law of Energy Conservation

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Is mechanical energy conserved when moving down an incline? If so, under what conditions is all of the potential energy converted to kinetic energy, leading to the greatest speed at the bottom?

Prediction

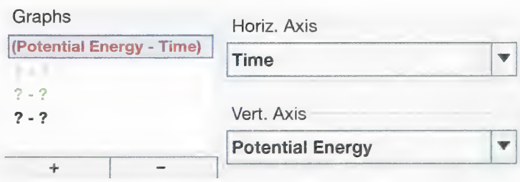
According to the law of conservation of energy, in an isolated system (frictionless), all of the gravitational potential energy of an object at the top of an incline should become kinetic energy when that object reaches the bottom of the incline. In a non-isolated system, when friction is present, energy will be lost; therefore, the total mechanical energy will decrease over time as the amount of potential energy available to be converted to kinetic energy decreases. The applet used in the following simulation helps you explore the motion and forces acting on a mass moving down a smooth, frictionless plane.

Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms "inclined planes" in the search bar. Choose the item called "Inclined Plane: Frictionless." Open the simulation. You can learn more about the simulation and how to use it by reading the Show Me found at the top of the simulation screen.

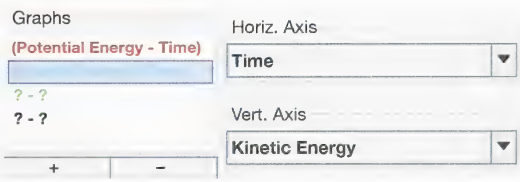
Procedure and Observations

On the simulation, select the "View Graph" tool (graph). Then complete the following steps:

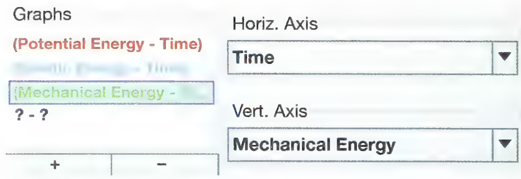
- Click on the first set of question marks ("? - ?"). Choose "Time" for the horizontal axis and "Potential Energy" for the vertical axis.



- Click on the next available set of question marks. Choose "Time" for the horizontal axis and "Kinetic Energy" for the vertical axis.



- Click on the next available set of question marks. Choose "Time" for the horizontal axis and "Mechanical Energy" for the vertical axis.

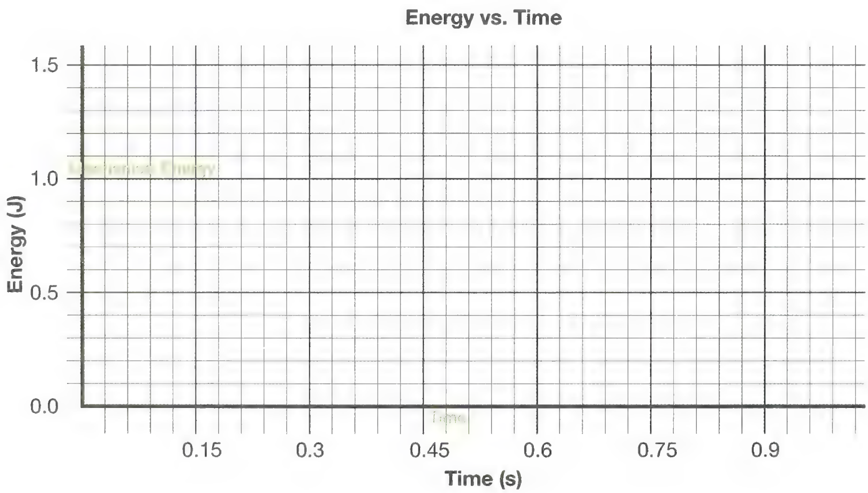


Check to ensure that the coefficient of friction is set to zero.

Press "Play," and watch the changes in potential and kinetic energy. On the graph, select the "Fit graph" tool to make the graph fully visible. Drag the margins to the right and down to enlarge the window. Click on the down arrow just under the "Zoom In" button to expand the graph further downward.

Self-Check

SC 1. Sketch the graph for potential energy, kinetic energy, and mechanical energy versus time.



Check your work with the answer in the appendix.

Analysis

Module 6: Lesson 2 Assignment

Remember to submit the answers to LAB 1 and LAB 2 to your teacher as part of your Lesson 2 Assignment.

LAB 1.

a. What is the total change in potential energy for the block when there is no friction present?

b. Where has this energy gone? How can you use this to find the velocity of the block as it reaches the bottom of the incline?

c. Calculate the velocity of the block at the bottom of the incline. Use the weight of the block from the data shown at the top of the simulation and the energy from the first graph.

LAB 2. Assume that the block has a potential energy of 0 J when it reaches the bottom of the incline. What can be said about the sum (mechanical energy) of the potential and kinetic energy for the block at any point on the way down the incline when no friction is present?

Read

Read "Mechanical Energy" on pages 306 to 309 of your textbook.

Self-Check

SC 2. Complete question 2 of "Practice Problems" on page 309 of your textbook.

Check your work with the answer in the appendix.

Try This

TR 1. Solve using energy considerations. Recall from the previous lesson that $E_p = \frac{1}{2}kx^2$. Since $F = kx$, $E_p = \frac{1}{2}Fx$.

a. A rubber-band slingshot shoots a 25-g stone. What is the initial speed of the stone if the rubber band is drawn back 0.15 m with a force of 27 N?

b. How high will the stone rise if it is shot straight upward?

Read

Read about mechanical energy in isolated systems on pages 311 to 314 of your textbook.

Self-Check

SC 3. Complete question 1 of "Practice Problems" on page 313 of the textbook.

Check your work with the answer in the appendix.

Module 6: Lesson 2 Assignment

Remember to submit the answer to TR 2 to your teacher as part of your Lesson 2 Assignment.

Try This

TR 2. Solve these questions using energy considerations.

a. A 0.80-kg coconut is growing 10 m above the ground in its palm tree. The tree is just at the edge of a cliff that is 15 m tall. What would the maximum speed of the coconut be if it fell to the ground beneath the tree?

b. What would the maximum speed be if it fell from the tree to the bottom of the cliff?

Read

Read about mechanical energy in pendulums on pages 314 to 316 of the textbook.

Self-Check

SC 4. Complete question 1 of "Practice Problems" on page 315 of the textbook.

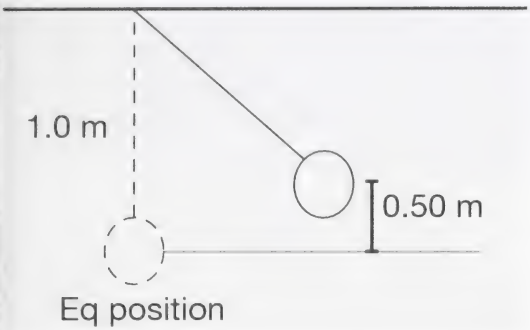
Check your work with the answer in the appendix.

Module 6: Lesson 2 Assignment

Remember to submit the answer to TR 3 to your teacher as part of your Lesson 2 Assignment.

Try This

TR 3. A pendulum is dropped from the position as shown in the diagram (0.50 m) above the equilibrium position. What is the speed of the pendulum bob as it passes through the equilibrium position?



Energy Changes in Non-Isolated Systems

Up to now in this lesson, we have dealt with a frictionless environment. What happens when we consider mechanical energy systems that include friction? Complete the rest of the lab to see.

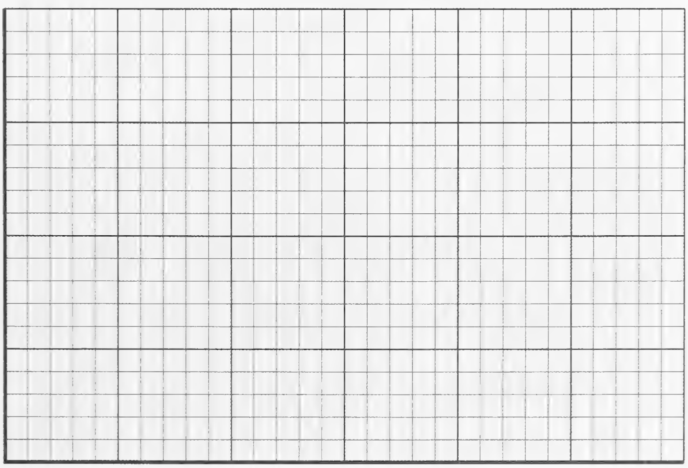
Module 6: Lesson 2 Assignment

Remember to submit the answer to LAB 3 to your teacher as part of your Module 6: Lesson 2 Assignment.

LAB 3. If necessary, re-open the "Inclined Plane: Frictionless" simulation.

Set the coefficient of friction to 0.2, and repeat the procedure. You may have to click "Zoom Out" several times to see the whole graph. Sketch the three graphs on the grid below. Label the lines and axes.

Energy vs. Time



Repeat several trials with greater coefficients of friction each time, up to a maximum of 0.55. Observe how the mechanical energy changes in each case.

Self-Check

SC 5. Describe the relationship between the coefficient of friction and the total mechanical energy as the object slides down the incline.

Check your work with the answer in the appendix.

Module 6: Lesson 2 Assignment

Remember to submit the answers to LAB 4 and LAB 5 to your teacher as part of your Module 6: Lesson 2 Assignment.

LAB 4.

a. What is the total change in potential energy for the block when there is friction present and the coefficient of friction is 0.20?

b. Has all this energy become kinetic? If not, where did it go?

LAB 5. Assume that the block has a potential energy of 0 J when it reaches the bottom of the incline.

a. What can be said about the sum (mechanical energy) of the potential and kinetic energy for the block at any point on the way down the incline when friction is present?

b. Is mechanical energy still conserved?

Conclusion

The total mechanical energy (sum of the potential and kinetic energy) is conserved in an isolated environment. Any change in either potential or kinetic energy is associated with an equal but opposite change in the other energy. As an object slides down an incline, potential energy is converted into kinetic energy until all of the energy has been transferred from one form to the other.

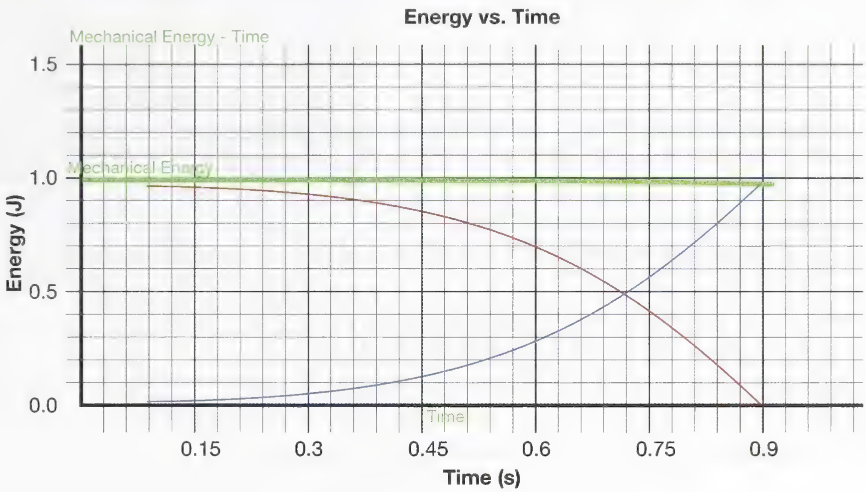
Mechanical energy is not conserved when friction is present, making the system non-isolated. In this case, some of the potential energy is converted to heat due to the friction, rather than being converted to kinetic energy. Therefore, the mechanical energy decreases over time. By reducing the friction, the loss in mechanical energy can be minimized, resulting in a greater conversion of potential energy into kinetic energy and a greater speed at the bottom of an incline.

- In a frictionless (isolated) environment, mechanical energy is conserved. Therefore, E_p at the top equals E_k at the bottom.
- In a friction (non-isolated) environment, E_p at the top equals E_k at the bottom plus work done to overcome friction.

It is important to note that the forces related to potential energy (gravitational and elastic forces) and kinetic energy act within the isolated system and do not affect the mechanical energy of the system. These forces are called **conservative forces**. Other forces, such as friction and forces from outside a non-isolated system, affect the mechanical energy of the system and are called **non-conservative forces**. Go to your Physics 20 Multimedia DVD, and choose the item called "Conservative Forces and Non-conservative Forces." The simulation gives a visual and graphical representation of these forces.

conservative force: a force such as gravity, acting in an isolated system, where the total work done is independent of the path an object is moved through

non-conservative force: a force acting on a non-isolated system from outside the system or from friction; a force where the total work done depends on the path an object is moved through



Reflect and Connect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Boarding down the half-pipe is an excellent example of energy transfers between potential energy and kinetic energy. With each jump, kinetic energy is converted to potential energy and then back again as the jump is landed. The process repeats itself as long as the rider has energy to continue, or until the bottom of the pipe is reached. This does not mean, however, that the mechanical energy is conserved. In fact, by the time the boarder reaches the bottom of the hill, he or she needs to have lost all the mechanical energy he or she had at the top of the hill. Therefore, this is a good example of a non-isolated system.

How do you think the snowboarder controls the rate at which potential energy is converted into kinetic energy?

How does the snowboarder ensure that all the mechanical energy he or she had, with respect to the top of the hill, has been lost?

What would happen to the snowboarder at the bottom of the hill if the mechanical energy were to be conserved—that is, if the half-pipe were frictionless?

Store the answers to these questions in your Physics 20 course folder.

Discuss


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What is the relationship between work and the conservation of mechanical energy? In an isolated system, total mechanical energy is conserved, with no energy leaving or entering the system. In a non-isolated system, energy can leave or enter, which means that work is being done. In the skiing and snowboarding examples shown in this lesson, energy was leaving the system as work done by friction.

In the discussion forum, describe a non-isolated system where energy “enters,” leading to an *increase* in mechanical energy. Describe how kinetic and potential energy can be transferred in the system and why work must be done to add energy to the system.

Reflect on the Big Picture

Each of the Reflect on the Big Picture sections in this module will ask you to consider gravitational, frictional, or elastic forces. To help you consolidate your learning from this lesson, complete at least one of these reflection activities:

- Explain the physics idea of work to a Grade 6 student who is watching a construction crane hold a heavy box 6 m in the air. Do you expect that the student will be receptive to the idea that no work is being done on the box?

- Imagine what it would be like if you could bounce on a trampoline forever and there would never be any loss of energy. Since the eighth century, people have been working on perpetual motion machines. So far, no such machine has been built. Do you think such a machine will ever be built? Prepare a short list of problems that would have to be overcome.

Store your completed reflection in your Physics 20 course folder.

Lesson Summary

In this lesson you explored the following questions:

- Is mechanical energy conserved in an isolated (frictionless) and non-isolated (friction) environment?

- What effect does the presence of friction have when converting potential energy into kinetic energy?

- What effect does work have on mechanical energy?

The total mechanical energy (sum of the potential and kinetic energy) is conserved in an isolated environment. Any change in either potential or kinetic energy is associated with an equal but opposite change in the other energy. As an object slides down an incline, potential energy is converted into kinetic energy until all of the energy has been transferred from one form to the other. The forces acting in an isolated system are called conservative forces.

Mechanical energy is not conserved when friction is present, making the system non-isolated. In this case, some of the potential energy is lost due to the work done by friction rather than being converted to kinetic energy. Therefore, the mechanical energy decreases over time. By reducing friction, the loss in mechanical energy can be minimized, resulting in a greater conversion of potential energy into kinetic energy and a greater speed at the bottom of an incline. Forces acting on a non-isolated system from outside the system or from friction are called non-conservative forces.

In a frictionless (isolated) environment, mechanical energy is conserved. Therefore, E_p at the top equals E_k at the bottom. In a friction (non-isolated) environment, E_p at the top equals E_k at the bottom plus work done by friction.

Lesson Glossary

conservative force: a force such as gravity, acting in an isolated system where the total work done is independent of the path an object is moved through

law of conservation of energy: a principle that states that within an isolated system, energy cannot be created or destroyed, but it can be converted from one form to another

mechanical energy: the sum of potential energy and kinetic energy

non-conservative force: a force acting on a non-isolated system from outside the system or from friction; a force where the total work done depends on the path an object is moved through

Lesson 3—Mechanical Energy and the Work-Energy Theorem

Get Focused

Downhill skiing in the mountains is made easier by the chairlift. Without it, very few people would ever have a chance to experience skiing at altitudes high enough to be in the clouds. Think about how much fun you would have if you had to hike up the mountainside—it would be a lot of work packing yourself and all your ski gear up the side of mountain in waist-deep snow!

© Gary & Sandy Wales/shutterstock

While some people do choose other ways to get up to the top of the mountain, most skiers choose the chairlift as their energy source. It works against Earth's gravitational field, moving uphill at a constant speed, from the bottom to the top. It also works against the force of friction, which acts whenever the chairlift is in operation. Given these facts, how does the work done by the chairlift relate to the gain in the mechanical energy of the skiers and the work done to overcome friction?

In this lesson you will explore the following questions:

- How do you know how much work is done by a chairlift?
- What effect do conservative and non-conservative forces have on the gain in mechanical energy of the skiers on the lift?
- What is the work-potential energy theorem?

Module 6: Lesson 3 Assignments

Your Lesson 3 Assignment in the Module 6 Assignment Booklet requires you to submit a response to the following:

- Lab—LAB 1, LAB 2, LAB 3, LAB 4, LAB 5, LAB 6, LAB 7, LAB 8, LAB 9, LAB 10, and LAB 11
- Try This—TR 1
- Discuss—D 1 and D 2

You must decide what to do with the questions that are not marked by the teacher.

Remember that these questions provide you with the practice and feedback that you need to successfully complete this course. You should respond to all the questions and place those answers in your course folder.

Explore

You will recall from Module 6: Lesson 2 that when a skier descends or comes down from the top of the mountain, gravitational potential energy is converted to kinetic energy—eventually, all the mechanical energy is lost due to friction. At the bottom of the hill, when standing still, the mechanical energy of the skier is zero relative to the mechanical energy he or she had at the top of the hill. This means that mechanical energy is not conserved; therefore, energy must be added in order to continue skiing once the skier reaches the bottom of the hill. Enter the chairlift.

When the skier gets back on the chairlift at the bottom of the mountain, work is done by the lift against the force of gravity as it moves back up the mountain. Some of the work done by the lift is transferred to the skier in the form of mechanical energy, and the whole process starts again. Some of the work done by the chairlift, however, is lost. Understanding how to measure work and compare it to potential energy gains in the presence and absence of friction will help you understand this process.

Graphical Representation of Work

What is work? When an object, such as a skier, is lifted at a constant velocity, work is being done against the force of gravity. Recall the earlier definition of work from Module 6: Lesson 1.

Work is a measure of the amount of energy transferred when a force acts over a given displacement. It is the product of the magnitude of the applied force and the displacement of the object in the direction of that force. Expressed as an equation, it is

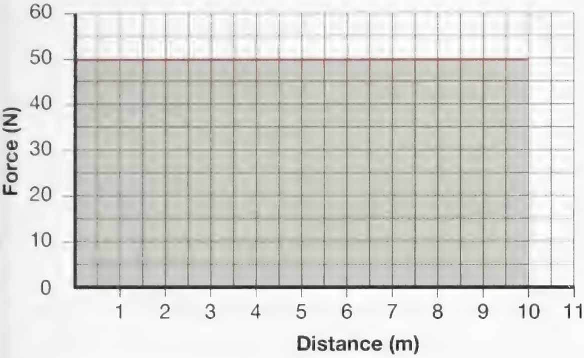
$$W = F_{\parallel} \Delta d$$

This is often written as $W = F\Delta d$.

Quantity	Symbol	SI Unit
work	W	joule (J or N•m)
force (the component of the force that is parallel to the displacement of the object)	$F_{\parallel}$	newton (N)
displacement (in the same direction as force)	d	metre (m)

1 J is equal to 1 N•m. Work is the product of the magnitudes of two vector quantities. Hence, it is a scalar quantity.

The equation $W = F_{\|}\Delta d$ is easily represented graphically as *the area under a force-distance graph* because the area is equal to the product of force and distance. For example, if a 50.0-N force acts through a distance of 10.0 m, how much work is done by the force?



$W = F_{\|}\Delta d$

$W = \text{area}$

$W = (50.0\ \text{N})(10.0\ \text{m})$

$W \doteq 500.00\ \text{N}\bullet\text{m}$

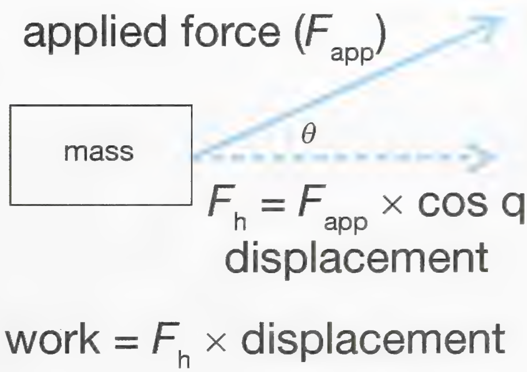
$W = 500\ \text{J}$ correct to 3 significant digits

Understanding the Component of Force That Does the Work

In the equation $W = F_{\|}\Delta d$, $F_{\|}$ represents the component of the force that acts in the *same* direction as the motion (it is parallel to the motion) and Δd represents the change in position. In many cases, however, the force and displacement are not in the same direction.

Recall that component analysis is used to find the work done when the force is not in the same direction as the displacement.

In these situations, the work equation becomes $W = F\cos(\theta)\Delta d$, where θ is the angle between the direction of the force and the direction of the change in position (Δd).



If the angle between F and Δd is not constant, it will be necessary to recalculate $W = F\cos(\theta)\Delta d$ for each value of θ and Δd and then add the results. When you do this, you still find that the area under a force-displacement graph equals the work done by that force.

Now you will use this fact to investigate the relationship between the work done and the gain in mechanical energy when an object is lifted in the presence of conservative forces (gravity) and non-conservative forces (friction).

Lesson 3 Lab: Work-Potential Energy Theorem

Problem

What is the relationship between the work done and the gain in mechanical energy for an object that is moved vertically in Earth's gravitational field?

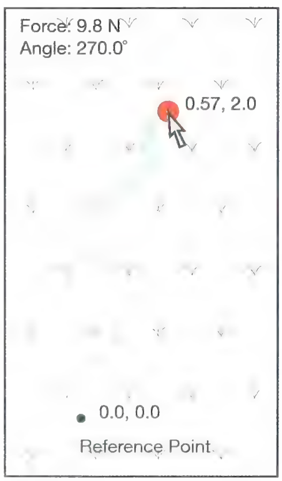
Module 6: Lesson 3 Assignment

Remember to submit the answers to LAB 1, LAB 2, LAB 3, LAB 4, and LAB 5 to your teacher as part of your Lesson 3 Assignment.

Procedure and Observations

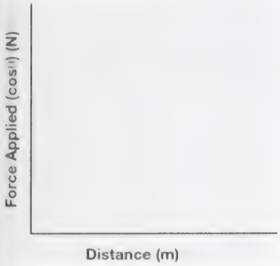
Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms "work-energy theorem" in the search bar. Choose the item called "Work-Energy Theorem." Complete the following steps:

- Select the "Force Field" (☑ Force Field) at the top of the window to display the gravitational force vectors. (These are in the direction of the force you must do work against—in this case, gravity.)
- Click and hold on the reference point and drag out a path until the y component (second value) of the position reads 2.0 as shown at the right.



- Once the path is drawn, click the "View Graph" tool (graph). You may have to drag the bottom border of the window down to see the buttons. On the horizontal axis, select "Distance"; on the vertical axis, select "Applied Force * cos (Theta)."

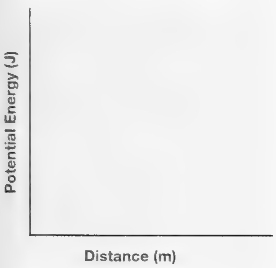
LAB 1. Click "Fit graph" tool, and sketch the graph on your answer sheet.



LAB 2. Continue with the simulation by completing the following steps:

Enlarge the graph by dragging the right-hand side of the graph window to the right till you can see the last two buttons clearly. Determine the area (work) of the force-distance graph by pressing the "Integrate the Selected Graph" tool ( ) and dragging out the area under the graph from right to left. The area will be shown under the word "Output" at the top of the screen (Integral:). Record the area on your answer sheet.

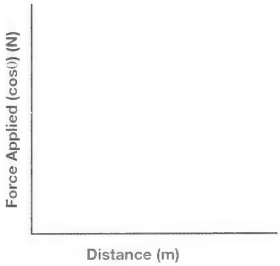
LAB 3. Change the vertical axis of the graph to "Potential Energy," and click the "Fit graph" tool button. Sketch the graph below. Click the "Options" button ( ). Select "Generate Table" to see the potential energy at the end of the motion. You can change the size of the columns in the data table by clicking on the title and dragging to the right. Record the potential energy at the end with the work done on the force-distance graph.

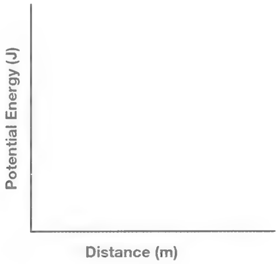


work (area) = __________ potential energy at the end = ___________

LAB 4. According to your results in LAB 1, LAB 2, and LAB 3, does the work done equal the gain in potential energy when work is done against a conservative force such as gravity? In this context, what does the term *conservative* mean?

LAB 5. Reset the simulation, and close the graph window. Repeat the steps outlined at the beginning of the procedure using a different path that ends up at the same spot as the one completed in LAB 1 ($y = 2.0$). **Note:** You have to delete any old graphs before making new ones.





work (area) = ________ potential energy at the end = ________

Analysis

Self-Check

SC 1. Based on your graphs above, if the work done equals the change in potential energy, why doesn't the length of the path you take to move the object the same vertical height matter? (**Hint:** Recall that work is only done when the force and motion are in the same direction.)

Check your work with the answer in the appendix.

Module 6: Lesson 3 Assignment

Remember to submit the answer to LAB 6 to your teacher as part of your Lesson 3 Assignment.

Conclusion

There is a very simple relationship between the work that you do when you lift an object through a known distance and the change in that object's potential gravitational energy.

LAB 6. Work done is equal to the change in _______________.

Read

There is a very simple relationship between the work that you do when you lift an object through a known distance and the change in that object's potential gravitational energy. The gravitational force is known as a *conservative force*. (From Lesson 2, you may remember that conservative force acts in an isolated system, such as gravity, where the total work done is independent of the path an object is moved through.) Path independence is a very important property of conservative forces. This means that when an object does work against a conservative force, the work done does not depend on the path taken. The work done depends only on the final change in position or the vertical displacement of the object (since the force applied is vertical when you lift an object).

For example, since you know that the gravitational force is conservative, you can easily calculate the work done in lifting an object from one position to another. You don't have to worry at all about the path that the object takes. As you showed earlier, the work done is equal to the change in potential energy.

This is written as

$$W = \triangle E_p$$
$$W = mg\triangle h$$

In the formula, Δh is the change in height. This is sometimes called the **work-potential energy theorem**.

If the object being worked on also gains kinetic energy, the work done is defined as

$$W = \triangle E_k + \triangle E_p$$

work-potential energy theorem: a statement that says the work done in a gravitational field is equal to the change in potential energy

work-energy theorem: a statement that says the work done on a closed system is equal to the sum of the changes in the potential and kinetic energies of the system

This is known as the **work-energy theorem**—the work done on a closed system is equal to the sum of the changes in the potential and kinetic energies of the system.

If a chairlift moves a skier vertically upwards in a frictionless environment, the work done by the chairlift will be equal to the potential energy gain of the skiers when they reach the top of the mountain. However, the chairlift is not a closed system. In other words, energy may leave in the form of heat and sound generated by friction forces. Such forces are called *non-conservative forces*. The force of friction, for example, will act along the entire path of the chairlift from top to bottom.

Lesson 3 Lab: Work-Energy Theorem Lab

Problem

Is the work-potential energy theorem applicable when a non-conservative resistance force, such as friction, is present?

Procedure

Re-open the Work-Energy Theorem simulation, if necessary. Complete the following steps:

- Reset the simulation and close the graph window.
- Turn on the non-conservative friction force by selecting the “Resistance” (☑ Resistance) option.
- Create a path by clicking on the reference point and dragging the object upwards.
- Use the graphing techniques that you used in the Work-Potential Energy Lab to measure the work done, the change in potential energy, and the total length of the path (distance). **Note:** You have to delete any old graphs before making new ones. To do this, use the “Add” and “Subtract” buttons on the grapher (+ | -).

Module 6: Lesson 3 Assignment

Remember to submit the answers to LAB 7, LAB 8, LAB 9, and LAB 10 to your teacher as part of your Lesson 3 Assignment.

Observations

LAB 7. In the following table, summarize your findings for three different paths that end at approximately the same spot.

Path	Change in Height (m)	Work Done (J)	Change in Potential Energy (J)	Length of Path (m)
1				
2				
3				

Analysis

LAB 8. Based on your data above, does path independence hold when there are non-conservative resistance forces present?

LAB 9. How does the work done compare with the change in potential energy when non-conservative forces are present?

LAB 10. Is the work-potential energy theorem applicable when there are resistive or non-conservative forces present?

Module 6: Lesson 3 Assignment

Remember to submit the answer to LAB 11 to your teacher as part of your Lesson 3 Assignment.

Conclusion

LAB 11. Fill in the blanks. If we think of the forces as a combination of conservative and non-conservative parts, then we can say that

work done = ________________________ + _________________________.

Read

Read "Conservative and Non-conservative Forces" on pages 319 and 320 of your textbook.

Self-Check

SC 2.

a. Name two conservative forces and one non-conservative force.
b. How does the force of friction affect the mechanical energy of the system?

Check your work with the answer in the appendix.

Read

Read "Energy Changes in Non-isolated Systems" on pages 321 and 322 of the textbook.

Self-Check

SC 3. Complete question 7 of "6.3 Check and Reflect" on page 323 of the textbook.

Check your work with the answer in the appendix.

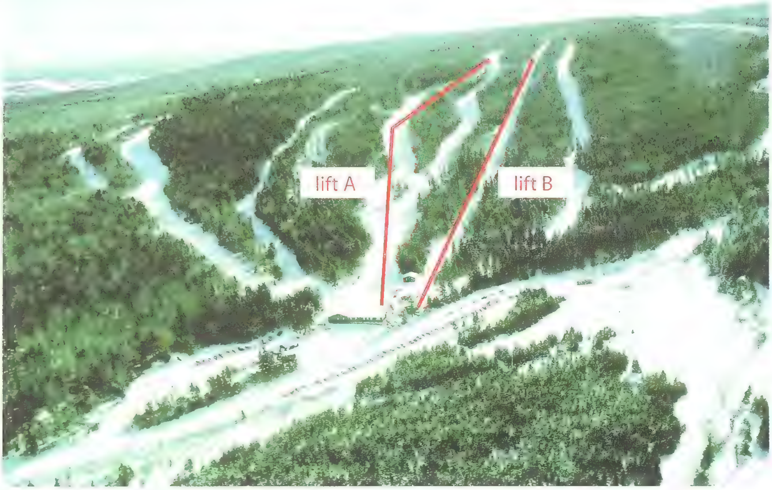
Module 6: Lesson 3 Assignment

Remember to submit the answer to TR 1 to your teacher as part of your Lesson 3 Assignment.

Try This

TR 1. Complete question 8 of "6.3 Check and Reflect" on page 323 of your textbook.

Reflect and Connect



There are two chairlifts in operation at the local ski hill. Both lifts move riders up the same vertical distance.

Which chairlift would be more economical to operate? Why?

Do the skiers that ride lift A have any more energy at the top than the riders on lift B?

Since lifts A and B move riders the same vertical distance, skiers (of equal mass) getting off of each chairlift will have the same mechanical energy. Therefore, in the presence of a conservative force, such as gravity, the work done to raise the skiers on both lifts will be identical and equal to their respective mechanical energies when they reach the top. Path independence exists in this situation, so both lifts would require the same amount of energy to operate.

Chairlifts, however, are not closed systems. When a non-conservative force, such as friction, is present, work must be done to overcome it. In this case, the longer path will require more work since friction must be overcome along the entire length of the path. Therefore, more work must be done to move a skier from the bottom to the top on lift A. The extra amount of work is to overcome the non-conservative force of friction. This makes lift B much more economical than lift A.

Module 6: Lesson 3 Assignment

Remember to submit the answer to the D 1 and D 2 to your teacher as part of your Lesson 3 Assignment.

Discuss

In the discussion forum, propose an explanation for the following facts. Include a description of the energy transfers that would take place when the lift starts and stops.

D 1. If the chairlift engine fails, a brake system is required to prevent the chairlift from reversing directions.

D 2. A fully loaded chairlift, as seen here, requires more energy to start than it does to maintain its motion.

© Nick Stubbs/shutterstock

Reflect on the Big Picture

Each of the Reflect on the Big Picture sections in this module will ask you to consider gravitational, frictional, or elastic forces. To help you consolidate your learning from this lesson, complete at least one of these reflection activities:

- Take, find, or sketch a picture of a skier on a ski lift. There are many forces acting to keep the skier from reaching the top of the ski run. Mark the picture to represent these forces. How do these forces affect the work the ski lift has to do to take the skier to the top of the run?
- Prepare a list of at least five websites that deal with the work-energy theorem. Rank the websites on the basis of how useful they would be for a beginning physics student.

Store your completed reflection in your Physics 20 course folder.

Module 6: Lesson 3 Assignment

Make sure you have completed all of the questions for the Lesson 3 Assignment. Check with your teacher about whether you should submit your assignment now or wait until all of the Module 6 assignments have been completed.

Lesson Summary

In this lesson you explored the following questions:
- How do you know how much work is done by a chairlift?
- What effect do conservative and non-conservative forces have on the gain in mechanical energy of the skiers on the lift?
- What is the work-potential energy theorem?

In the presence of a conservative force (gravity), the work done in raising an object, such as a skier, through a vertical distance is equal to the gain in potential energy of that object. This is known as the work-potential energy theorem.

In the presence of both a non-conservative force (friction) and a conservative force (gravity), the work done raising an object vertically is equal to the sum of the gain in potential energy and the work done to overcome the non-conservative force. For example, the work done by a chairlift is not equal to the gain in mechanical energy of the riders since some energy is lost working against the force of friction. The longer the pathway, the more energy that is lost to the non-conservative force.

Lesson Glossary

work-energy theorem: a statement that says the work done on a closed system is equal to the sum of the changes in the potential and kinetic energies of the system

work-potential energy theorem: a statement that says the work done in a gravitational field is equal to the change in potential energy

Lesson 4—Work and Power

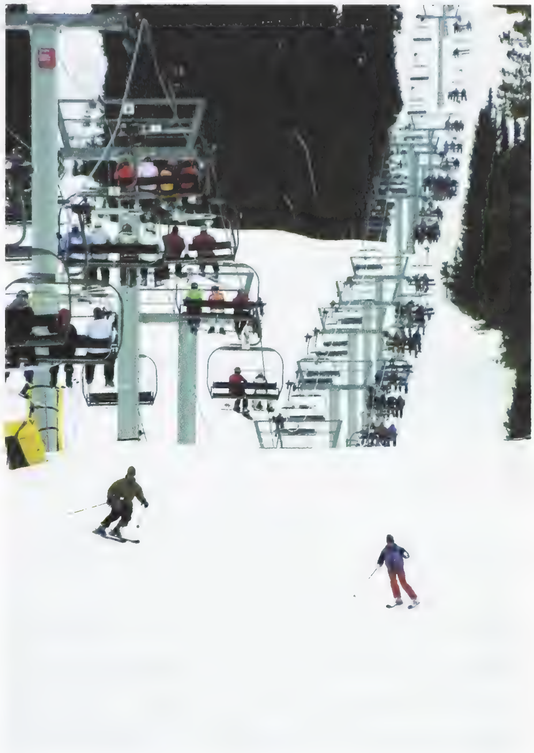
Get Focused

Have you ever been on a chairlift, when it suddenly stopped halfway up the hill? Has this ever happened to you when it was cold and windy? When it's windy and you're dangling from a chair high above the ground, things can get very uncomfortable very quickly. The less time you spend hanging on the chairlift, enduring winter weather, the better.

Have you ever been skiing on your winter holidays? If so, you have likely witnessed large lineups for the lifts. During these times, people spend more time in line and less time enjoying potential-kinetic energy transfers (skiing).

Is there a common solution to both of these problems?

In considering the safety risks with people enduring bitter cold while suspended high above the ground and the frustration of skiers waiting in lineups, it became evident that a new kind of chairlift was needed—a "high-speed chair." As the name clearly suggests, this chairlift simply goes faster than the older chairlifts, moving riders up the hill at a faster rate. The result is that individuals spend less time waiting for, and riding, the chairlift and more time skiing. Everybody wins. Or do they?



The ski-hill operator has to pay for electricity to operate the lift. This cost is passed on to the customers when they purchase lift tickets. If the new high-speed chair replaces an older, slower chairlift and follows the same route as the original, will it cost more or less money to operate? The answer to this depends on how much power is required to operate the lift and on its efficiency.

In this lesson you will explore the following questions:

- What is power?
- What is the relationship between power and work?
- How do we determine efficiency?

Module 6: Lesson 4 Assignment

Your Lesson 4 Assignment in the Module 6 Assignment Booklet requires you to submit a response to the following:

- Try This—TR 1, TR 2, TR 3, and TR 4
- Discuss—D 1 and D 2

You must decide what to do with the questions that are not marked by the teacher.

Remember that these questions provide you with the practice and feedback that you need to successfully complete this course. You should respond to all the questions and place those answers in your course folder.

Watch and Listen

Other than speed, what is different in a faster ski lift? Go to your Physics 20 Multimedia DVD, and watch the chairlift simulation titled "The Relationship Between Power and Work."

Explore

Power

In the previous simulation, you will have noticed that when the lift is operating at any speed, the riders are gaining gravitational potential energy as soon as they are loaded at the bottom. Recall that a rider's gain in gravitational potential energy is equal to the work done against the conservative force of gravity. Since the lift is not a closed system, energy will also be needed to overcome non-conservative forces, such as friction. Therefore, the work done by a chairlift can be defined as follows:

work done = energy to overcome conservative forces + energy to overcome non-conservative forces

$$W = E_{\mathrm{p}} + E_{\mathrm{lost}}$$

power: the rate of doing work

If you assume that the E_{lost} due to non-conservative forces, such as friction, is constant, regardless of the speed at which the chairlift operates, is the work done by the lift constant, regardless of how fast it moves? If E_{p} and E_{lost} are constant for a given mass, regardless of the speed at which it moves, the work done is not related to its speed. The **power** required to do the work at different speeds, however, does change. In the previous simulation, you should have noticed that when the lift operates at a high rate of speed, it consumes more power. What this means is that it does the same amount of work in lifting each rider of equal mass, but it does it faster and, in the process, consumes more power. Therefore, power is defined by the amount of work done in a given time interval.

Expressed as an equation, power is

$$P = \frac{W}{\Delta t} \text{ or } \frac{\Delta E}{\Delta t}$$

Quantity	Symbol	SI Unit
power	P	W
work	W	J
change in time	t	s

1 W = 1 J/s of work. The unit of power was named in recognition of the scientific contributions of James Watt. Another common reference to power is "horsepower" (1 hp = 746 W).

Why does the chairlift require more power to move faster? You can answer this mathematically by substituting the equation for work into the power equation and then rearranging and simplifying it.

$$P = \frac{W}{\Delta t}$$
$$P = \frac{F \Delta d}{\Delta t}$$
$$P = F\left(\frac{\Delta d}{\Delta t}\right)$$
$$P = F\, v_{\text{ave}}$$

Therefore, you can see that if the conservative and non-conservative forces are constant, increasing the power acting on the system will cause an increase in the average speed of the system.

Read

Read "Power, Work, and Time" on pages 324 and 325 of the textbook to see examples of these concepts being used.

Self-Check

SC 1. Complete question 1 of "Practice Problems" on page 325 of the textbook.

Check your work with the answer in the appendix.

Module 6: Lesson 4 Assignment

Remember to submit the answer to TR 1 to your teacher as part of your Lesson 4 Assignment.

Try This

TR 1. A garage door opener is rated at 373 W. How much work can it do in 14.0 s of operation?

Read

What if a power question gives speed instead of time? Read “Power and Speed” on pages 327 to 329 in your textbook to see the answer.

Self-Check

SC 2. Complete question 3 of “Practice Problems” on page 328 of the textbook.

Check your work with the answer in the appendix.

Module 6: Lesson 4 Assignment

Remember to submit the answers to TR 2 and TR 3 to your teacher as part of your Lesson 4 Assignment.

Try This

TR 2. An automobile engine exerts a force of 595 N to propel the car at 90.0 km/h on a level road. What is the power output of the engine at that time?

TR 3. A traditional, double-seat chairlift can move 1200 people per hour (with an average mass of 60.0 kg) up a ski hill. A high-speed quad chair can move 2400 people in 30.0 minutes up the hill. If the upper terminal is 100 m higher than the lower terminal, how much power would each type of lift need just to overcome the force of gravity? Explain why they would need more power than the amount used to overcome the force of gravity.

Efficiency

Could a high-speed chairlift do more work using the same energy as the slower, older chairlift consumed? If so, it would be considered more efficient. In other words, less of the work done by the lift would be wasted by non-conservative forces (E_{lost}) while still providing the same amount of energy for overcoming the conservative forces (E_p).

To determine the **efficiency** of the chairlift, you must first identify the input and output energy for the chairlift. Recall that the work done by the lift is transferred to the potential energy of the riders and the energy lost due to non-conservative forces such as friction. Therefore, $W_{in} = E_p + E_{lost}$.

efficiency: the ratio of the output (useful work) to the input (total energy used)

Expressed as an equation, it is

$$\text{Efficiency} = \frac{\text{Energy output}}{\text{Energy input}} \text{ or } \frac{\text{Power output}}{\text{Power input}}$$

Since the input and output occur in the same time interval, efficiency can be determined using either power or energy.

In this equation, W is the work done by the lift—the energy input—and E_p is the conserved gravitational potential energy—the output. Therefore, efficiency could be described as follows:

$$\text{efficiency} = \frac{E_p}{W}$$

$$\text{efficiency} = \frac{E_p}{\left(E_p + E_{lost}\right)}$$

From this equation, you can see that if E_{lost} is reduced, efficiency will increase. Perfect efficiency would only be achieved if E_{lost} becomes zero, which is the case in a closed system (where all the work done becomes mechanical energy). The chairlift, however, is an open system, so it has less than 100% efficiency. If you assume that the lost energy (E_{lost}) is nearly the same, regardless of speed, both the slow and fast chairlifts have identical efficiency.

So what is the net effect of increasing the speed of a chairlift? If the high-speed lift is just as efficient as the slow-speed lift, why does the high-speed lift use more power? On each type of lift, E_{lost} is assumed to be identical, regardless of the speed. If the high-speed chair consumes more power in a time interval, the extra work done by the high-speed lift results in a greater amount of gravitational potential energy. But how can you have more potential energy without going farther up the hill? The answer is, move more mass. If the high-speed and low-speed lifts operate at capacity for the same time interval, the high-speed lift will have moved much more mass up the hill.

Increasing the speed of the chairlift will require more power and will move more riders per day when operating at capacity. The required power is purchased, which increases the costs of operating the ski hill. Additional lift-pass sales or an increase in the cost per pass (or both) would be required to support paying for the lifts capable of higher speeds, but patrons would be able to spend more time skiing and less time sitting or waiting in line.

Self-Check

SC 3. What is the efficiency of a lift that uses 5.1 kW of power to move 24 people of average mass 60 kg a vertical distance of 80 m in 15 min?

Check your work with the answer in the appendix.

Module 6: Lesson 4 Assignment

Remember to submit the answer to TR 4 to your teacher as part of your Lesson 4 Assignment.

Try This

TR 4. Complete question 6 of "6.4 Check and Reflect" on page 330 of the textbook.

Reflect and Connect

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If a new high-speed, quad chairlift replaces a slower, double chairlift, will it cost more to operate? Assuming that the energy lost due to non-conservative forces, such as friction, is identical for both lifts, the high-speed lift is still more costly to operate at the same capacity. The high-speed lift requires more power to run at higher speeds. However, it will transport more riders per day while exposing skiers to less severe winter weather since they spend less time on the chair and more time skiing. The following table compares the two types of lifts.

Low-Speed Double Chair	High-Speed Quad Chair
• low-power operation equals less power purchased by the operator • requires light infrastructure (cables, towers) • moves 600 riders per hour at 2.5 m/s	• more power needed to operate equals more power purchased by the operator • requires heavier infrastructure (cables, towers) • moves 2400 riders per hour at 5.0 m/s

Now, consider the fact that most lifts do not operate at full capacity. If fewer riders are on the lift, and the power supply is not adjusted, the lift would increase in speed. With more riders and a constant power supply, the lift would slow down. In order to prevent this, the power supplied to the lift cable is adjusted to maintain the speed of the lift as the mass of riders getting on and off changes. This is done to increase efficiency and ensure most of the work done by the motors is converted to the gravitational potential energy of the riders. If the power is not adjusted according to the number of riders on the lift, the kinetic energy of the system would increase and more energy would be lost due to friction. This, in turn, leads to higher operating costs.

Module 6: Lesson 4 Assignment

Remember to submit the answers to D 1 and D 2 to your teacher as part of your Lesson 4 Assignment.

Discuss

In the discussion forum, answer the following two questions.

D 1. Assumptions have been made in this lesson that the energy lost operating a low-speed chairlift is the same as the energy lost operating a high-speed lift of equal path length. Why is this assumption false? What impact does this have on the efficiency of the lift?

D 2. How could efficiency be maximized on a chairlift that can have anywhere from 0 to 2400 riders per hour?

© Douglas Litchfield/shutterstock

Reflect on the Big Picture

Each of the Reflect on the Big Picture sections in this module will ask you to consider gravitational, frictional, or elastic forces. Complete at least one of these reflection activities:

- Imagine that you are the owner of a new ski resort. Search the Internet for at least two manufacturers of ski lifts. From the information you find and what you have learned in this unit, decide which ski lift you will buy. To support your choice, build a chart or other graphical presentation of the ski lifts and write a short summary explaining the choice you have made. Be sure to consider energy efficiencies in your summary.

- Imagine that you are the owner of a new ski resort. In a world with increasing energy costs, using efficient equipment might be the difference between business success and business failure. Prepare a list of decisions that you will have to make over the next few years that relate to energy efficiency.

Store your completed reflection in your Physics 20 course folder.

Module 6: Lesson 4 Assignment

Make sure you have completed all of the questions for the Lesson 4 Assignment. Submit the Module 6 Assignment Booklet to your teacher.

Lesson Summary

In this lesson you explored the following questions:

- What is power?
- What is the relationship between power and work?
- How is efficiency determined?

Power is the rate of doing work. It is the ratio of the work done to the time interval in which it is done. In the case of the chairlift, the power does work on the riders, increasing their mechanical energy during the time they are moving up the hill. The power is also consumed doing work against non-conservative force, such as friction. More power is required to do more work. Increasing the power on a chairlift will cause it to move at a faster rate, moving more mass up the hill in less time.

Efficiency is the ratio of the output (useful work) to the input (total energy used). For a chairlift, the useful work is equal to the gain in potential energy of the riders once they reach the top of the lift. The total energy used is greater than this since some of the energy is consumed overcoming non-conservative forces such as friction. Minimizing non-conservative forces will lead to greater efficiency.

Lesson Glossary

efficiency: the ratio of the output (useful work) to the input (total energy used)

Expressed as an equation, it is

$$\text{Efficiency} = \frac{\text{Energy output}}{\text{Energy input}} \text{ or } \frac{\text{Power output}}{\text{Power input}}$$

Since the input and output occur in the same time interval, efficiency can be determined using either power or energy.

power: the rate of doing work

Module 6 —Work and Energy

Module Summary

Work is a measure of the amount of energy transferred when a force acts over a given displacement. It is the product of the magnitude of the applied force and the displacement of the object in the direction of that force.

You have examined three forms of energy and their interactions in isolated and non-isolated environments:

- Gravitational potential energy is the energy of an object due to its position above Earth's surface.

- Elastic potential energy is energy produced or consumed by an object, which is altered from its standard shape, without permanent deformation.

- Kinetic energy is the energy due to the motion of an object.

A bungee jump begins with gravitational potential energy. On the way down, the energy is converted to kinetic energy and finally to elastic potential energy at the very bottom. On the way up, the process is reversed. With each bounce, some energy is lost from the system and eventually the motion stops.

Total mechanical energy (the sum of the potential energy and the kinetic energy) is conserved in an isolated environment. Any change in either potential or kinetic energy is associated with an equal but opposite change in the other energy. As an object slides down an incline, potential energy is converted into kinetic energy until all of the energy has been transferred from one form to the other. The forces acting in an isolated system are called conservative forces.

Mechanical energy is not conserved when friction is present, making the system non-isolated. In this case, some of the potential energy is lost due to the work done by friction rather than being converted to kinetic energy. Therefore, the mechanical energy decreases over time. By reducing the friction, the loss in mechanical energy can be minimized, which results in a greater conversion of potential energy into kinetic energy and a greater speed at the bottom of an incline. Forces acting on a non-isolated system from outside the system or from friction are called non-conservative forces.

- In a frictionless (isolated) environment, mechanical energy is conserved. Therefore, E_{p} at the top equals E_{k} at the bottom.

- In a friction (non-isolated) environment, E_{p} at the top equals E_{k} at the bottom plus work done by friction.

In the presence of a conservative force (gravity), the work done in raising an object, such as a skier, through a vertical distance is equal to the gain in potential energy of that object. This is known as the work-potential energy theorem.

In the presence of both a non-conservative force (friction) and a conservative force (gravity), the work done raising an object vertically is equal to the sum of the gain in potential energy and the work done to overcome the non-conservative force. For example, the work done by a chairlift is not equal to the gain in mechanical energy of the riders since some energy is lost working against the force of friction. The longer the pathway, the more energy is lost to the non-conservative forces.

Power is the rate of doing work. It is the ratio of the work done to the time interval in which it is done. In the case of the chairlift, the motor does work on the riders, increasing their mechanical energy during the time they are moving up the hill. The energy created by the motor is also consumed doing work against non-conservative forces, such as friction. To counteract friction, more power is needed. Increasing the power (doing more work in the same time period) on a chairlift will cause it to move at a faster rate, moving more mass up the hill in less time.

Efficiency is the ratio of the energy output (useful work) to the energy input (total energy used). For a chairlift, the useful work is equal to the gain in potential energy of the riders once they reach the top of the lift. The total energy used is greater than this since some of the energy is consumed overcoming non-conserved forces, such as friction. Minimizing non-conserved forces will lead to greater efficiency.

Module 6 Assessment

You are expected to submit the following items to your teacher for marks:

- Module 6: Lesson 1 Assignment
- Module 6: Lesson 2 Assignment
- Module 6: Lesson 3 Assignment
- Module 6: Lesson 4 Assignment

Choose one of the Reflect on the Big Picture items from this module, and share it with your teacher.

Unit C Conclusion

In Module 5 you saw that uniform circular motion is a special case of two-dimensional motion. An examination of uniform motion revealed the relationships among speed, frequency, period, and radius for circular motion; and it became evident that acceleration is directed toward the centre of a circle. You applied Newton's laws of motion to help explain uniform circular motion, and then you used circular motion to approximate elliptical orbits and explain planetary, natural, and artificial satellite motion. You were able to predict the mass of a celestial body from the orbital data of a satellite in uniform circular motion around that body. Finally, you examined the relationship between Kepler's laws and the development of Newton's universal law of gravitation.

At the beginning of this module you were asked, "What conditions are necessary to maintain circular motion?" Throughout the module, you gained understanding to help you answer this question. When an object travels in a circular path, there must be an inward force causing the direction of the motion to change—so, if there is circular motion, there is also a centripetal force. Free-body analysis can be used to visualize the forces acting on an object, and the equation for centripetal force helps you understand the conditions that are necessary to maintain circular motion.

Module 6 examined the relationships among kinetic, gravitational potential, and total mechanical energies of a mass at any point between maximum potential energy and maximum kinetic energy. You analyzed kinematics and dynamics problems that related to the conservation of mechanical energy in an isolated system, and you looked at the change in mechanical energy in a system that is not isolated. This helped you to understand the difference between conservative and non-conservative forces. You came to see that while work is a measure of the mechanical energy transferred, power is the rate of doing work. You then applied your knowledge of work and power to solve related problems.

In an isolated system, no energy is lost but, rather, is transferred from gravitational potential to kinetic and back. You saw this with the examples of skiers using a chairlift. However, in a system that is not isolated, energy can be lost to friction, heat, and sound. Energy can be gained from motors and other outside sources. Even if energy is lost from the system, it is not destroyed—it is merely transformed. Energy that has been gained has not been created but, rather, transferred from an outside source into the system. This knowledge is used to design more efficient energy transfer systems in society.

Unit C Assessment

A 0.160-kg ball attached to a light cord is swung in a vertical circle of radius 70.0 cm. At the top of the swing, the speed of the ball is 3.26 m/s. The centre of the circle is 1.50 m above the floor.

a. Draw a free-body diagram of the forces on the ball at the top of the swing.

b. Calculate the magnitude of the tension in the cord at the top of the swing.

c. Calculate the mechanical energy of the ball at the top of the swing with respect to the floor.

d. Calculate the speed of the ball when the cord is 30.0° down from horizontal.

e. Determine the magnitude of the tension in the cord when the cord is 30.0° down from horizontal.

Submit your answers to your teacher for marks.

Module Glossary

conservative force: a force such as gravity, acting in an isolated system where the total work done is independent of the path an object is moved through

efficiency: the ratio of the output (useful work) to the input (total energy used)

Expressed as an equation, it is

$$\text{Efficiency} = \frac{\text{Energy output}}{\text{Energy input}} \text{ or } \frac{\text{Power output}}{\text{Power input}}$$

Since the input and output occur in the same time interval, efficiency can be determined using either power or energy.

elastic potential energy: energy stored in a spring or elastic object that has been compressed or stretched

gravitational potential energy: the potential energy of an object due to its height above Earth

Hooke's law: the amount of stretch (deformation) of an elastic object is proportional to the force applied to deform it

kinetic energy: energy that a body has because of its motion

law of conservation of energy: a principle that states that within an isolated system, energy cannot be created or destroyed, but it can be converted from one form to another

mechanical energy: the sum of potential energy and kinetic energy

mechanical system: a system that has both potential and kinetic energy

non-conservative force: a force acting on a non-isolated system from outside the system or from friction; a force where the total work done depends on the path an object is moved through

power: the rate of doing work

work: the energy transferred by a force to a moving object; the product of a force and the distance through which the force is applied

work-energy theorem: a statement that says the work done on a closed system is equal to the sum of the changes in the potential and kinetic energies of the system

work-potential energy theorem: a statement that says the work done in a gravitational field is equal to the change in potential energy

Self-Check Answers

Lesson 1

SC 1.

Given

$m = 25.0$ kg $d = 6.0$ m $t = 4.0$ s $F_f = 3.8$ N

Required

the work done in moving the object (W)

Analysis and Solution

The applied force will be the force to accelerate the object plus the force to overcome friction. The acceleration is calculated as follows:

$$d = v_i \Delta t + \tfrac{1}{2} a (\Delta t)^2 \text{ but } v_i = 0\text{, so}$$
$$d = \tfrac{1}{2} a (\Delta t)^2$$
$$a = \frac{2d}{(\Delta t)^2}$$
$$a = \frac{2 \times 6.0 \text{ m}}{(4.0 \text{ s})^2}$$
$$a = 0.75 \text{ m/s}^2$$

The net force can be calculated as follows:

$$F_{net} = m a$$
$$F_{net} = (25.0 \text{ kg})(0.75 \text{ m/s}^2)$$
$$F_{net} \doteq 18.75 \text{ N}$$

The applied force is calculated as follows:

$$F_{net} = F_{app} + F_f$$
$$F_{app} = F_{net} - F_f$$
$$F_{app} = 18.75 \text{ N} - (-3.8 \text{ N})$$
$$F_{app} \doteq 22.55 \text{ N}$$

The work done is calculated as follows:

$$W = F\Delta d$$
$$W = (22.55\ \text{N})(6.0\ \text{m})$$
$$W \doteq 135.3\ \text{J}$$
$$W = 1.4 \times 10^2\ \text{J correct to 2 significant digits}$$

Paraphrase

There was 1.4×10^2 J of work done in moving the object.

SC 2.

Given

$m_e = 750$ kg
$m_1 = 65.0$ kg
$m_2 = 30.0$ kg
$m_3 = 48.0$ kg
$h = -21.0$ m

Required

the change in gravitational potential energy (ΔE_p)

Analysis and Solution

Find the total mass of the elevator and the three people. Use the formula for change in gravitational potential energy to find the required answer.

$m_t = 750\ \text{kg} + 65.0\ \text{kg} + 30.0\ \text{kg} + 48.0\ \text{kg}$
$m_t = 893$ kg

$\Delta E_p = mg\Delta h$
$\Delta E_p = (893\ \text{kg})(9.81\ \text{m/s}^2)(-21.0\ \text{m})$
$\Delta E_p = -1.84 \times 10^5$ J

Paraphrase

The change in gravitational potential energy of the car and its passengers is
-1.84×10^5 J.

SC 3.

Given

$m = 250$ kg
$\Delta d = 20.0$ m
$\theta = 35.0°$

Required

the change in potential energy (ΔE_p)

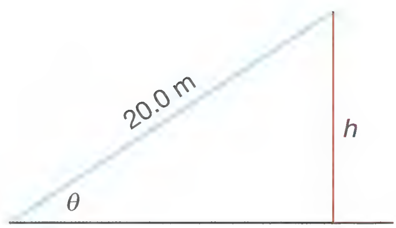
Analysis and Solution

Choose the height of the bottom of the incline as the reference point. Find the height of the incline, and use the gravitational potential energy formula.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin\theta = \frac{h}{20.0\ \text{m}}$$

$$h = (\sin 35.0°)(20.0\ \text{m})$$
$$h = 11.47\ \text{m}$$



$$\Delta E_p = mg\Delta h$$
$$= (250\ \text{kg})(9.81\ \text{m/s}^2)(11.47\ \text{m})$$
$$= 2.81 \times 10^4\ \text{J}$$

Paraphrase

The change in potential energy is 2.81×10^4 J.

SC 4.

(a)

Given

$E_p = 500$ J
$x = 0.400$ m

Required

the force to produce the stretch (F)

Analysis and Solution

Use the elastic potential energy formula to find k. Then use Hooke's law to find the force.

$$E_p = \tfrac{1}{2}kx^2$$

$$k = \frac{2E_p}{x^2}$$

$$k = \frac{2(500\ \text{N•m})}{(0.400\ \text{m})^2} \qquad F = kx$$

$$F = (6250\ \text{N/m})(0.400\ \text{m})$$

$$k = 6250\ \text{N/m} \qquad F = 2.50 \times 10^3\ \text{N}$$

Paraphrase

The force needed to produce the stretch is 2.50×10^3 N.

(b)

Given

$F = 1000$ N
$x = 0.400$ m

Required

the change in elastic potential energy (ΔE_p)

Analysis and Solution

Use Hooke's law to find the stretch, and insert that value into the elastic potential energy formula to find the new potential energy. Then subtract the original potential energy from the new value to get the change in potential energy.

$$E_p = \frac{1}{2}kx^2$$

$$F = kx \qquad E_p = \frac{1}{2}(6250\ \text{N/m})(0.160\ \text{m})^2$$

$$x = \frac{F}{k} \qquad E_p = 80\ \text{J}$$

$$x = \frac{1000\ \text{N}}{6250\ \text{N/m}} \qquad \Delta E_p = 80\ \text{J} - 500\ \text{J}$$

$$x = 0.160\ \text{m} \qquad \Delta E_p = -420\ \text{J}$$

Paraphrase

The change in elastic potential energy is −420 J.

SC 5.

Given

$m_g = 45.0$ kg
$m_b = 16.0$ kg
$v = 2.50$ m/s

Required

the kinetic energy of the system (E_k)

Analysis and Solution

Find the total mass of the girl and bicycle system. Use the kinetic energy formula to find the answer.

$$\begin{aligned} m_t &= m_g + m_b \\ m_t &= 45.0\text{ kg} + 16.0\text{ kg} \\ m_t &= 61.0\text{ kg} \end{aligned}$$

$$\begin{aligned} E_k &= \frac{1}{2}mv^2 \\ E_k &= \frac{1}{2}(61.0\text{ kg})(2.50\text{ m/s})^2 \\ E_k &= 191\text{ J} \end{aligned}$$

Paraphrase

The kinetic energy of the system is 191 J.

SC 6.

(a)

Given

$k = 2500$ N/m
$x = 0.540$ m

Required

the elastic potential energy stored in the bow (E_p)

Analysis and Solution

Use the elastic potential energy formula to find the answer.

$$\begin{aligned} E_p &= \frac{1}{2}kx^2 \\ &= \frac{1}{2}(2500\ \text{N/m})(0.540\ \text{m})^2 \\ &\doteq 364.5\ \text{J} \\ &= 365\ \text{J correct to three significant digits} \end{aligned}$$

Paraphrase

The elastic potential energy stored in the bow is 365 J.

(b)

Given

$m = 95.0$ g
$E_p = 365$ J

Required

the speed of the arrow (v)

Analysis and Solution

Convert the mass of the arrow to kg.

$$95.0\ \text{g} \times \frac{1\ \text{kg}}{1000\ \text{g}} = 0.0950\ \text{kg}$$

All of the potential energy is converted to kinetic energy. Rearrange the kinetic energy formula to solve for the speed.

$$E_k = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2(364.5\text{ J})}{0.0950\text{ kg}}} = 87.6\text{ m/s}$$

unit cancelation check:

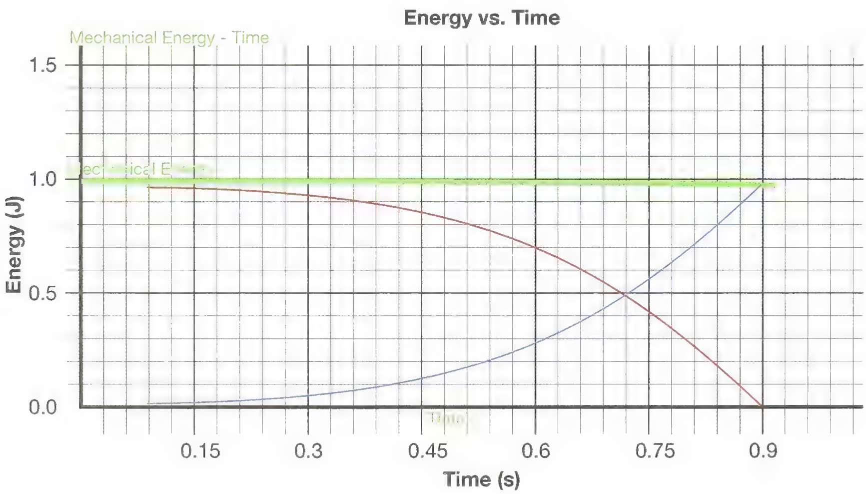
$$\sqrt{\frac{\text{J}}{\text{kg}}} = \sqrt{\frac{\text{N}\bullet\text{m}}{\text{kg}}} = \sqrt{\frac{\text{kg}\cdot\text{m/s}^2\bullet\text{m}}{\text{kg}}} = \text{m/s}$$

Paraphrase

The speed of the arrow is 87.6 m/s.

Lesson 2

SC 1.



SC 2.

Given

$m = 4.50$ kg
$Em = 6.27 \times 10^4$ J
$h = 275$ m

Required

the speed of the cannon ball (v)

Analysis and Solution

The cannon ball will possess potential energy because of its height and kinetic energy as a result of its speed. Find speed from the kinetic energy using the total mechanical energy formula.

$E_m = E_k + E_p$
$E_k = E_m - E_p$

$$\frac{1}{2}mv^2 = E_m - mgh$$

$$v = \sqrt{\frac{2E_m}{m} - 2gh}$$

$$v = \sqrt{\frac{2(6.27 \times 10^4 \text{ J})}{4.50 \text{ kg}} - 2(9.81 \text{ m/s}^2)(275 \text{ m})}$$

$$v = 150 \text{ m/s}$$

Paraphrase

The speed of the cannon ball is 150 m/s.

SC 3.

Given

$E_{ki} = 250$ J
$\Delta E_p = 650$ J (loss)

Required

the final kinetic energy (E_{kf})

Analysis and Solution

The ramp is frictionless, so this is an isolated system. According to the law of conservation of energy, the loss of potential energy will equal the gain in kinetic energy. Therefore, the final kinetic energy will equal the initial kinetic energy less the change in potential energy.

$\Delta E_k = -\Delta E_p$
$E_{kf} - E_{ki} = -\Delta E_p$
$E_{kf} = E_{ki} - \Delta E_p$
$E_{kf} = (250 \text{ J}) - (-650 \text{ J})$
$E_{kf} = 900$ J

Paraphrase

The final kinetic energy of the crate is 900 J.

SC 4.

Given

$h_f = 15.0$ cm
$h_i = 25.0$ cm
$m = 250$ g

Required

(a) the mechanical energy (E_m)
(b) the kinetic energy (E_{kf})
(c) the speed (v)

Analysis and Solution

(a) The pendulum is frictionless. So this is an isolated system, and mechanical energy will be conserved. The mechanical energy at any point is equal to the initial potential energy because there was no kinetic energy at the start.

$E_{mi} = E_{pi} + E_{ki}$
$E_{mi} = mgh_i + ½mv_i^2$
$E_{mi} =(0.250 \text{ kg})(9.81 \text{ m/s}^2)(0.250 \text{ m}) + ½(0.250 \text{ kg})(0 \text{ m/s})^2$
$E_{mi} = 0.613$ J

(b) According to the law of conservation of energy, the loss of potential energy will equal the gain in kinetic energy. Therefore, the kinetic energy will equal the initial mechanical energy less the change in potential energy.

$E_{kf} = E_{mf} - \Delta E_{pi}$
$E_{kf} = E_{mf} - mg\Delta h$
$E_{kf} = (0.613 \text{ J}) - (0.250 \text{ kg})(9.81 \text{ m/s}^2)(0.250 \text{ m} - 0.15 \text{ m})$
$E_{kf} = 0.245$ J

(c) Use the kinetic energy formula to find the speed.

$E_{kf} = \frac{1}{2}mv_f^2$

$$v_f = \sqrt{\frac{2E_{kf}}{m}}$$
$$v_f = \sqrt{\frac{2(0.245\ \text{J})}{0.25\ \text{kg}}}$$
$$v_f = 1.40\ \text{m/s}$$

Paraphrase

(a) The mechanical energy is 0.613 J.
(b) The kinetic energy is 0.245 J.
(c) The speed is 1.40 m/s.

SC 5. As the coefficient of friction increases, the total mechanical energy as the object slides down the incline decreases.

Lesson 3

SC 1.

Since there are only vertical forces acting, no work is done in the horizontal direction. Therefore, the path length may be extended in the horizontal direction with no impact on the work done or the gain in potential energy.

SC 2.

a. Two conservative forces are gravity and elastic forces within the system, and one non-conservative force is friction.

b. The force of friction reduces the mechanical energy of the system.

SC 3.

Given

$m_A = 2.40$ kg
$m_B = 1.50$ kg
$\Delta h = 1.40$ m

Required

- an explanation as to why this is or is not an isolated system
- the kinetic energy of this system (E_{kf})

Analysis and Solution

The system can be considered an isolated system because the pulley is frictionless and air resistance is negligible, so there are no non-conservative forces. As mass A falls, its loss in potential energy will be totally converted to a gain in potential energy of mass B and a gain in kinetic energy of the system.

$$\begin{aligned}
\Delta E_{p_A} &= \Delta E_{p_B} + \Delta E_k \\
\Delta E_k &= \Delta E_{p_A} - \Delta E_{p_B} \\
\Delta E_k &= m_A g \Delta h - m_B g \Delta h \\
\Delta E_k &= (m_A - m_B) g \Delta h \\
\Delta E_k &= (2.40\text{ kg} - 1.50\text{ kg})(9.81\text{ m/s}^2)(1.40\text{ m}) \\
\Delta E_k &= 12.4\text{ J}
\end{aligned}$$

Paraphrase

The kinetic energy of the system the instant before mass A hits the tabletop is 12.4 J.

Lesson 4

SC 1.

Given

$m = 1.50$ t
$h = 65.0$ m
$t = 3.50$ min

Required

the power in watts and hp (P)

Analysis and Solution

The work done by the engine of the crane increases the gravitational potential energy of the mass. Convert the mass to kilograms and the time to seconds as part of the calculations in the equations.

$$P = \frac{\Delta E_p}{t}$$

$$P = \frac{mg\Delta h}{t}$$

$$P = \frac{\left(1.50\ \text{t} \times \frac{1000\ \text{kg}}{1\ \text{t}}\right)\left(9.81\ \text{m/s}^2\right)\left(65.0\ \text{m}\right)}{\left(3.50\ \text{min} \times \frac{60\ \text{s}}{1\ \text{min}}\right)}$$

$$P = 4.55 \times 10^3\ \text{W}$$

$$P = 4.55 \times 10^3\ \text{W} \times \frac{1\ \text{hp}}{746\ \text{W}}$$

$$P = 6.11\ \text{hp}$$

Paraphrase

The power of the engine is 4.55 kW or 6.11 hp.

SC 2.

Given

$m = 1250$ kg
$v_i = 0.00$ m/s
$v_f = 30.0$ m/s
$t = 4.00$ s

Required

the power output of the motor (P)

Analysis and Solution

The work done is to accelerate the car, so calculate the acceleration and use Newton's second law to calculate the force required. Then use $P = F_{net} v_{ave}$ to calculate the power.

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$
$$\vec{a} = \frac{(30.0 \text{ m/s}) - (0.00 \text{ m/s})}{4.00 \text{ s}}$$
$$\vec{a} = 7.50 \text{ m/s}^2$$

$$\vec{F}_{net} = m\vec{a}$$
$$\vec{F}_{net} = (1250 \text{ kg})(7.50 \text{ m/s}^2)$$
$$\vec{F}_{net} = 9375 \text{ N}$$

$$P = F_{net} v_{ave}$$
$$P = (9375 \text{ N})\left(\frac{30.0 \text{ m/s}}{2}\right)$$
$$P = 1.41 \times 10^5 \text{ W}$$

Paraphrase

The power output of the motor is 1.41×10^5 W or 141 kW.

SC 3.

Given

$P_{in} = 5.1$ kW
$m = (24 \times 60 \text{ kg})$
$h = 80$ m
$t = 15$ min

Required

the efficiency of the lift

Analysis and Solution

Calculate the power output; then calculate the efficiency.

$$P_{out} = \frac{\Delta E_p}{t}$$

$$P_{out} = \frac{mg\Delta h}{t}$$

$$P_{out} = \frac{(24 \times 60 \text{ kg})(9.81 \text{ m/s}^2)(80 \text{ m})}{15 \text{ min} \times \frac{60 \text{ s}}{1 \text{ min}}}$$

$$P_{out} = 1255.68 \text{ W}$$

$$\text{Efficiency} = \frac{P_{out}}{P_{in}}$$

$$\text{Efficiency} = \frac{1255.68 \text{ W}}{5.1 \times 10^3 \text{ W}}$$

$$\text{Efficiency} = 0.25$$

$$\text{Efficiency} = 25\ \%$$

Paraphrase

The efficiency of the lift is 25%.